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Inflation Dynamics and Time-Varying Persistence: The Importance of the Uncertainty Channel

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Abstract

In this article, we employ a time-varying GARCH-type specification to model inflation and investigate the behaviour of its persistence. Specifically, by modelling the inflation series as AR(1)-APARCH(1,1)-in-mean-level process with breaks, we show that persistence is transmitted from the conditional variance to the conditional mean. Hence, by studying the conditional mean/variance independently, one will obtain a biased estimate of the true degree of persistence. Accordingly, we propose a new measure of time-varying persistence, which not only distinguishes between changes in the dynamics of inflation and its volatility but also allows for feedback between the two variables. Analysing the inflation series for a number of countries, we find evidence that inflation uncertainty plays an important role in shaping expectations, and a higher level of uncertainty increases inflation persistence. We also consider a number of unit root tests and present the results of a Monte Carlo experiment to investigate the size and power properties of these tests in the presence of breaks in the mean and the variance equation of an AR(1)-APARCH(1,1)-in-mean-level data generating process. The Monte Carlo experiment reveals that if the model is misspecified, then commonly used unit root tests will misclassify inflation as a nonstationary, rather than a stationary process.

Keywords: Inflation persistence, Conditional heteroscedasticity; GARCH-in-mean; unit root tests.

JEL Classification: E31; E58; C12; C22; C52.

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1 Introduction

Inflation persistence has been one of the most investigated topics in theoretical and applied works over the past decade. In the literature, there is not a single definition of persistence, however, in broad terms, the notion of persistence in economic models is closely related to the concept of inertia in physics. As Fuhrer (2010) points out, inertia may be defined as the resistance of a body to changing its velocity (direction and rate of speed) unless acted upon by an external force. In the case of inflation, the rate of change of the price level tends to remain constant in the absence of an economic "force" to move it from its current level. A closely related definition of persistence often found in the literature is the tendency of inflation to converge slowly towards its long-run value following a shock that has led inflation away from its long-run value (see, for example, Altissimo et al. 2006).

In theoretical models, inflation persistence is often explained using the hybrid New Keynesian Phillips curve, which relates the current inflation level to its own lags, the expectation of future inflation, the output gap, and real marginal costs. Each of these components of the Phillips curve is related to a specific source of inflation persistence. The first component is often referred to as "intrinsic persistence", and it's related to the backwards-looking behaviour of economic agents in re-setting prices (cost-push shocks) or negotiating wages. The second component is related to the so-called "expectations-based persistence", which is persistence due to the formation of inflation expectations. This type of persistence is related to asymmetric information of private agents' perceptions about the central bank's inflation target (see, for example, Cukierman and Meltzer, 1986; Tetlow and von zur Muehlen, 2001; Mankiw and Reis, 2002). The third component captures the lagged effect of the various macroeconomic shocks hitting inflation, such as persistent deviations of output from its potential level and it is often referred to as "extrinsic persistence" (see Angeloni et al., 2004).

To empirically study the effects of inflation persistence, researchers first need to be able to find a credible way of measuring it. In the literature, several persistence measures have been suggested. Conventional unit root tests have often been used to discriminate between stationary and non-stationary processes (see, for, example Gaglianone et al., 2018, Arize and Malindretos, 2012; Chen and Hsu, 2016). Other commonly used measures of persistence are the sum of the autoregressive (SUM) coefficients, the dominant root (or the largest AR root, LAR) or the half-life of innovations (e.g. Cogley and Sargent, 2001; Pivetta and Reis, 2007). Alternatively, measures of persistence employed in the literature exploit the idea that if inflation follows a mean-reverting process, it should cross its mean relatively frequently. Therefore, persistence is defined as the unconditional probability of a stationary stochastic process not crossing its mean in a given period (see Dias and Marques, 2010).

Against this background, the contribution of the paper to the ongoing debate is threefold. First, this study considers the issue of modelling the link between inflation and inflation uncertainty, emphasising the role of the transmission channel of economic shocks. Second, this work extends the literature on inflation persistence by proposing a new time-varying measure of persistence that accounts for the transmission of memory from the conditional variance to the conditional mean of the inflation process. Third, the paper investigates the performance of unit root tests in the presence of structural breaks. These tests are often used in empirical studies to investigate the behavior of inflation inertia in addition to the persistence measures mentioned above. We briefly present these contributions in turn below.

In relation to the first two points, existing literature on persistence focuses almost exclusively on the impact of persistence, but neglects the uncertainty channel. A voluminous literature has demonstrated that there is a close relationship between inflation level and its uncertainty (see Fountas et al., 2006; or Fountas and Karanasos, 2007). This field of research was pioneered by Friedman (1977), who argued that a rise in the average inflation rate leads to more uncertainty about future inflation. The author's argument is based on the viewpoint that uncertainty about future inflation distorts the allocative efficiency aspect of the price mechanism (for details, see, for example, Fountas et al., 2006; or Fountas and Karanasos, 2007). Following the influential work of Friedman, a rich literature has highlighted the importance of nominal uncertainty for macroeconomic modelling and policymaking. For example, using a repeated game between the public and the monetary authority, Ball and Cecchetti (1990) postulate that higher inflation results in higher inflation uncertainty. On the other hand, Cukierman and Meltzer (1986) argue that in the presence of uncertainty about the rate of monetary growth and, therefore, inflation, policymakers are inclined to apply expansionary monetary policy stances to surprise the agents and enjoy output gains. The argument that Central Banks tend to create inflation surprises in the presence of more inflation uncertainty implies a positive causal effect from inflation uncertainty to inflation and is closely related to the "expectations-based persistence" found in structural models.

While economic theory suggests that uncertainty may be a characteristic feature of inflation persistence in the literature, there are not many attempts at explicitly modelling this phenomenon. Existing empirical studies makes use of autoregressive-type models to estimate inflation persistence. This approach benefits easy implementation and close connection with the Solow-Tobin test of the natural rate hypothesis (see, for example, Fuhrer, 2010). However, in this paper, we argue that the measures of persistence mentioned above only capture the inertia due to the intrinsic component of the inflation formation mechanism, namely the extent to which economic agents look at the history of inflation when re-setting prices or negotiating wages (see for example Benati, 2008, Angeloni et al. 2003). In other words, the available reduced-form persistence measures limit the econometrician's ability to answer the question: "How long it will take for inflation to go back to its original level after a shock?" Here, our objective is to answer another, more challenging question: "What is the role of uncertainty on inflation persistence?". Also, has inflation persistence changed over time? Answering these questions is important since being able to disentangle the persistence that is deeply rooted in the economic structure and difficult to eradicate without incurring a recession from the persistence that can be controlled by well-managed inflation targeting policy is essential to policymakers.

The proposed time-varying measure is close in spirit to the SUM measure of persistence, and it is derived by reparametrizing a GARCH-in mean-type model into an ARMA(2,1) form to obtain an expression of the autoregressive coefficients that is a function of the estimated second moment of the inflation process. This expression can then be used to derive a measure that explicitly accounts for the persistence of uncertainty in addition to the intrinsic persistence. The advantage of the suggested measure is that it takes into account the inflation uncertainty by directly modelling the transmission of memory from the conditional variance to the conditional mean of the inflation stochastic process .

In contrast to related studies that mainly consider models with constant coefficients (see, for example, Fountas et al, 2006; Fountas and Karanasos, 2007; Grier et al., 2004), the GARCH-type model considered in this paper allows assessing if the inflation-inflation uncertainty relationship has changed over time. In particular, we postulate that the inflation process can be modelled using an AR(l)-APGARCH-in-mean-level (AR(l)-APGARCH-ML) specification, where the estimated parameters of the conditional mean and the conditional variance are allowed to change over time. This model was originally proposed by Engle et al. (1987) (see for example, Grier et al., 2004; Conrad et al., 2010; Conrad and Karanasos, 2010; and Karanasos and Zeng, 2013). However, in its original formulation, the model did not allow for structural breaks. This is a significant limitation when considering the inflation series, since a growing number of empirical works report that the inflation rate exhibits structural changes. For example, Sensier and van Dijk (2004) investigate the issue of structural changes in 214 US macroeconomic variables and show that even if several series experienced a break in conditional mean, most of the variation was due to changes in the conditional variance. Recent studies have shown that ignoring the presence of structural breaks can have important effects on the precision of inflation forecasting (see, for example, Caporale and Kontonikas, 2009; Caporale et al. 2010 and Chang and He, 2010).

Coming now to the third contribution of the paper, applied economists make use of unit root tests to classify inflation as either a stationary, I(0), or non-stationary, I(l), process. In most applied works, the first test of persistence is a unit root test, since if the inflation series is not stationary the process is not mean reverting and its variance is unbounded. However, the inflation series are characterised by autocorrelation structures that make it notoriously difficult to classify stochastic processes as I(0) or I(l). For example, in the literature, authors have reached different conclusions on the properties of the inflation time series for the US. Chandler and Polonik (2006), Beran (2009), and Palma and Olea (2010) find strong evidence for nonstationarity in the US inflation. On the other hand, Rose (1988) indicated that monthly U.S. inflation was an I(0) process from 1947 to 1986. Mixed evidence was provided by Brunner and Hess (1993). They concluded that the inflation rate was I(0) before the 1960s but that it is characterised as I(l) since then. Other studies include Narayan and Popp (2013), Gaglianone et al. (2018), and Kim et al. (2004), among others.

From the theoretical point of view, whether inflation follows a stationary or non-stationary process has important theoretical implications. In the literature, textbook treatments of inflation, such as Blanchard (2000) assume that inflation is stationary. Also, in their seminal paper, Blanchard and Gali (2007) suggest that inflation persistence captures structural characteristics of the economy that are not likely respond to policy actions, which implies that a policy of inflation targeting should exert no effect on inflation persistence. On the other side, the works by Cogley and Sargent (2005), Beechey and Osterholm (2009), and Cogley and Sbordone (2008) support the view that inflation persistence varies across monetary regimes, therefore do not support the Lucas critique. In this respect, it is well known that the performance of unit root tests depends on several factors that are not easily observed by applied researchers trying to discriminate between stationarity and nonstationarity. Standard unit root tests are based either on the assumption that the variance of the series is constant or on the assumption that some type of heteroscedasticity is present (e.g. Ling and Li, 2003; Ling et al., 2003; Rodrigues and Rubia, 2005; Kourogenis and Pittis, 2008) but do not consider the possibility that the volatility has a direct effect on the level. Also, a well-established literature has highlighted that these tests are not robust to structural breaks (see, for example, Iacone et al., 2021; Narayan and Popp, 2013). However, the performance of unit root tests in the presence of the in-mean parameter in the conditional mean equation is not well understood yet. Accordingly, we consider AR(l)-APGARCH-M data generating process and carry out an extensive Monte Carlo experiment to examine the size and power of these tests in the presence of abrupt breaks in the in-mean parameter.

The empirical results of this study reveal several insights into the dynamics of the inflation rate. First, we find evidence that the parameters in the models capturing intrinsic persistence and in-mean effects change over time. Therefore, not allowing for time-varying coefficients in the estimation procedure would result in less accurate modelling of the inflation process. Second, inflation uncertainty plays a vital role in shaping expectations, and a higher level of uncertainty increases inflation persistence. Finally, the Monte Carlo results indicate that the performance of commonly used unit root tests is severely affected by breaks. The above considerations reinforce the argument (and extend it to a dynamic environment) made by Canepa et al. (2019) that conventional time-invariant measures of persistence, such as unit roots, might result in misleading conclusions regarding the persistence in the level (see also Conrad and Karanasos, 2015; Canepa et al., 2020).

The outline of the paper is as follows. Section 2 introduces the AR(l)-APGARCH-ML model and the proposed measure of persistence. Section 3 presents the empirical analysis. Section 4 reports the results of the Monte Carlo simulation experiment. Section 5 discusses the monetary policy implications. Finally, Section 6 gives some concluding remarks.

2 The Model and Persistence Measure

The proposed measure of persistence relies on the estimation of an AR(1)-APGARCH(1, 1)-in mean-level effects (AR(1)-APGARCH(1,1)-ML), that is, a model in which the conditional variance affects the conditional mean and the level affects the conditional variance. We allow for deterministic abrupt breaks in the model. In particular, we examine the case of n breaks ($N = n$) that occur at time $t - k_1, ..., t - k_n$ (with $k_n > .... > k_1$, and $k_1, ..k_n \in \mathbb{Z}_{>0}$).

Let $\{y_t\}$ be the inflation process, the proposed model is given by

$$(1 - \phi(t) L)y_t = \varphi(t) + \varsigma(t)\sigma_t^{\delta} + \varepsilon_t, \tag{1}$$

where $\varepsilon_t = e_t\sigma_t$, L is the lag operator, $\delta > 0$, $\{e_t\}$ is a sequence of independent and identically distributed (*i.i.d*) random variables with zero mean and variance, $\mathbb{E}(e_t^2)$, and σ_t^2 is the conditional variance of y_t. The power transformed conditional variance, σ_t^{δ}, is positive with probability one and is a measurable function of $\mathcal{F}_{t-1}$, which in turn is the sigma-algebra generated by $\{y_{t-1}, y_{t-2}, \ldots\}$. We assume that the conditional variance equation is $\sigma_t^{\delta} \sim$ APGARCH$(1,1)$ given by

$$(1 - \beta(t) B)\sigma_t^{\delta} = \omega + \alpha(t) f(\varepsilon_{t-1}) + \varrho(t) y_{t-1}, \tag{2}$$

with

$$f(\varepsilon_{t-1}) = (|\varepsilon_{t-1}| - \gamma(t) \varepsilon_{t-1})^{\delta},$$

where $|\gamma(t)|$<1 for all t (for the APGARCH model with time invariant parameters see, for example, Ding *et al.*, 1993, and Karanasos and Kim, 2006). In Eq. (1) and (2) the vector of the deterministically time varying coefficients, $\mathbf{m}(\tau)' = (\varphi(\tau), \phi(\tau), \varsigma(\tau), \beta(\tau), \alpha(\tau), \varrho(\tau))$ is given by

$$\mathbf{m}(\tau)' = \begin{cases} (\varphi_1, \phi_1, \varsigma_1, \beta_1, \alpha_1, \varrho_1) & \text{if} \quad \tau > t - k_1, \\ (\varphi_2, \phi_2, \varsigma_2, \beta_2, \alpha_2, \varrho_2) & \text{if} \quad t - k_2 < \tau \leq t - k_1, \\ \quad\ldots & \quad\quad\ldots \\ (\varphi_n, \phi_n, \varsigma_n, \beta_n, \alpha_n, \varrho_n) & \text{if} \quad \tau \leq t - k_{n-1}. \end{cases} \tag{3}$$

with $\varphi_i, \phi_i, \varsigma_i, \beta_i, \alpha_i, \in \mathbb{R}$ (the set of real numbers), $i = 1, ..., n$, $\delta \in \mathbb{R}_{>0}$. [1] Note that, the process is weakly stationary if for $\sigma_t^{\delta} > 0$, for all t: $\omega > 0$, $\sum \beta_i + \sum \alpha_i < 1$, also the following conditions are necessary and sufficient for $\sigma_t^{\delta} > 0$, for all t: $\omega > 0$, $\alpha, \beta, \varrho \geq 0$ and $y_t \geq 0$ for all t. Finally, we denote the size of the breaks by $\Delta\phi_i = \phi_i - \phi_{i-1}$ with the breaks for the other parameters defined likewise.

According to (1) and (2) the breaks occur at times $t - k_1, ..., t - k_{n-1}$ and the switch from one set of parameters to another is abrupt. By including σ_t^{δ} in the conditional mean we allow for feedback from the power transformed conditional variance of y_t to its level, captured by the deterministically varying in-mean coefficient $\varsigma(t)$. Similarly, by including the lagged y_t in the conditional variance equation we allow for the time varying level effect. Therefore, the model allows for simultaneous feedback between the two variables.

Note that for $\delta \geq 1$, $\mu_{2/\delta} = \mathbb{E}(\sigma_t^2)$ is not a fractional moment only if δ is equal to 1 or 2. In all other cases $\mu_{2/\delta}$ has to be calculated numerically. However, if $\delta > 2$, the existence of the first moment, μ_1 guarantees that of $\mu_{2/\delta}$. Similarly, $\mu_{1+1/\delta} = \mathbb{E}(\sigma_t^{\delta+1})$ is not a fractional moment only if $\delta = 1/\lambda$ where $\lambda \in \mathbb{Z}_{>0}$. In all other cases $\mu_{1+1/\delta}$ has to be calculated numerically.

[1] Within the class of ARMA processes this specification is quite general and allows for intercept and slope shifts (see also Pesaran and Timmermann, 2005, Pesaran et al., 2006, and Koop and Potter, 2007).

The model in Eq. (1) and (2) can be estimated by Quasi-Maximum Likelihood Estimation method (QML). The asymptotic consistency of the QML estimator for the parametric GARCH-M model is established in Conrad and Mammen (2016).

2.1 Persistence Measure

Having described the model, we now exploit the dynamic properties of the specification in Eq.(1) and (2) to derive a parametric measure of persistence that accounts for both the impact of persistence of uncertainty and intrinsic persistence on the inflation process.

With this purpose in mind we reparameterise the model in Eq. (1) and (2) as time varying ARMA(2,1) model (for more details see Canepa et al., 2022; Conrad and Karanasos, 2015; Canepa *et al.*, 2019). Let κ_r denotes the r-th moment of $f(e_t)$: $\kappa_r = \mathbb{E}[[f(e_t)]^r]$, then Eq. (1) and (2) can be expressed as

$$\begin{aligned}(1 - g_1 L - g_2 L^2) y_t &= \varphi(t)^* + (1 - c(t) L)\varepsilon_t + \varsigma(t)\alpha(t) v_{t-1}, &(4)\\ g_1(t) &= \phi(t) + c(t) + \varsigma(t)\varrho(t), &(5)\\ g_2(t) &= -\phi(t) c(t), &(6)\\ \varphi^*(t) &= \varphi(t)(1 - c(t)), &(7)\end{aligned}$$

where

$$c(t) = \alpha(t)\kappa_1(t) + \beta(t), v_t = f(\varepsilon_t) - \mathbb{E}[f(\varepsilon_t)|\mathcal{F}_{t-1}] = f(\varepsilon_t) - \kappa_1(t)\sigma_t^\delta,$$

and v_t is, by construction, an uncorrelated term with expected value 0. While the ε_t are the innovations to the level of y_t, the v_t can be considered the 'innovations' to the power transformed conditional variance of y_t. In Eq. (4) the vector of the time-varying coefficients, $\mathbf{m}(\tau)'$, is defined as in Eq. (3).

Note that in Eq. (4), by including the lagged y_t in the conditional variance equation (the so-called level effect) and σ_t^δ in the mean equation (the so-called in-mean effect), we allow for simultaneous feedback between the two variables. The parameter $c(t)$ measures the *intrinsic* memory or persistence in the conditional variance (see also Conrad and Karanasos, 2015a). Note also that in Eq. (4) $g_1 > g_2$, by definition, since $\phi < 1$ implies that $|\phi(t) + c(t)| > |\phi(t) c(t)|$.

Next we will define the covariance matrix of the two 'shocks' ε_t and v_t, $\mathbf{\Sigma}_t = \mathbf{E}(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t')$, where $\boldsymbol{\varepsilon}_t = (\varepsilon_t \; v_t)'$ and $\mathbf{E}(\cdot)$ denotes the elementwise expectation operator. First, we will denote the variances of the two 'shocks' and their covariance by

$$\sigma_{\varepsilon t} = \mathbb{E}(\varepsilon_t^2), \sigma_{vt} = \mathbb{E}(v_t^2), \sigma_{\varepsilon v,t} = \mathbb{E}(\varepsilon_t v_t).$$

The covariance matrix $\mathbf{\Sigma}_t$ is given by

$$\mathbf{\Sigma}_t = \begin{bmatrix} \sigma_{\varepsilon t} & \sigma_{\varepsilon v,t} \\ \sigma_{\varepsilon v,t} & \sigma_{vt} \end{bmatrix} = \begin{bmatrix} \mu_{2/\delta,t}\mathbb{E}(e_t^2) & \mu_{1+1/\delta,t}\widetilde{\kappa}(t) \\ \mu_{1+1/\delta,t}\widetilde{\kappa}(t) & \mu_{2t}\kappa(t) \end{bmatrix}, \tag{8}$$

where

$$\kappa(t) = \kappa_2(t) - \kappa_1^2(t),\ \widetilde{\kappa}(t) = \mathbb{E}[e_t f(e_t)].$$

In a related work Canepa et al. (2022) show that under the assumption of Normality of the term e_t , the expressions for $\kappa_r(t)$ and $\widetilde{\kappa}(t)$ are given by

$$\begin{aligned}
\kappa_r(t) &= \frac{1}{\sqrt{\pi}}\left[(1-\gamma(t))^{r\delta} + (1+\gamma(t))^{r\delta}\right] 2^{(\frac{r\delta}{2}-1)}\Gamma\left(\frac{r\delta+1}{2}\right), \\
\widetilde{\kappa}(t) &= \frac{1}{\sqrt{2\pi}}\left[[1-\gamma(t)]^{\delta} - [1+\gamma(t)]^{\delta}\right] 2^{(\delta/2)}\Gamma\left(\frac{\delta}{2}+1\right),
\end{aligned}$$

where $\Gamma(\cdot)$ is the Gamma function.When $\delta = 1$ the above expressions reduce to $\widetilde{\kappa}(t) = -\gamma(t)$, $\kappa_1(t) = \sqrt{\frac{2}{\pi}}$, for all t, $\kappa_2(t) = 1 + \gamma^2(t)$ and therefore $\kappa(t) = \kappa_2(t) - \kappa_1^2(t) = 1 + \gamma^2(t) - \frac{2}{\pi}$, which implies that $\mathbf{\Sigma}_t$ becomes

$$\mathbf{\Sigma}_t = \mu_{2t}\begin{bmatrix} 1 & -\gamma(t) \\ -\gamma(t) & 1+\gamma^2(t) - \frac{2}{\pi} \end{bmatrix}. \tag{9}$$

From Eq. (4) it is easy to see that commonly used measures of persistence, such as the LAR or SUM obtained from the estimation of AR(p) models, are not able to account for persistence induced by the transmission of memory from the conditional variance to the conditional mean since in Eq. (4), by definition, $\phi < \phi + c(1-\phi)$. Also, from Eq. (4) it is easy to see that the inflation process may be highly persistent even if the intrinsic persistence in the level, ϕ, is low in magnitude. This is the case if c is large and the interaction effect between the inflation uncertainty and inflation (that is $\varsigma\gamma$, in Eq. (5)) is sufficiently strong.

To overcome these measures' possible shortcomings, we propose a time varying measure of persistence that accounts for the joint effect of the inflation intrinsic persistence and persistence due to its uncertainty. The suggested measure, denoted by $\pi(t)$, is closely related to the SUM measure. It involves replacing the sum of the coefficients in the autoregressive process with the parameters in Eq. (4). In this formulation $\pi(t)$ is given by

$$\pi(t) = \phi(t) + (1-\phi(t))c(t) + \varsigma(t)\varrho(t). \tag{10}$$

Note that in Eq. (10) $\pi < 1$, since under Assumption 1, the ARMA(2,1) process is covariance stationary. Also, for a given c, a large intrinsic persistence parameter ϕ reduces impact of persistence due to uncertainty, whereas if ϕ is small the impact of c will be large. In the limit, if $\phi \to 0$, then $(1-\phi(t)) \to 1$ and the persistence of the conditional variance will be fully transmitted to the conditional mean. Obviously, for a given ϕ, the inflation persistence will be higher if the persistence in the conditional variance is greater (i.e. π is increasing in c). Finally, in Eq. (10) the impact of inflation level on uncertainty, measured by $\varrho(t)$, also plays a role on inflation persistence via the joint effect measured by the interaction parameter $(\varsigma(t)\varrho(t))$. Therefore, the impact of $\varrho(t)$ will be stronger if the magnitude of

$\varsigma(t)$ is greater, that is if the transmission of the memory from the conditional variance accounted by $\varsigma(t)$ is greater.

Summing up, the most important feature of the proposed measure of persistence is that the expression in Eq. (10) allows for persistence of uncertainty that is related to imperfect information of economic agents about the nature of economic shocks. In the literature, this uncertainty is often related to gradual responses of inflation to shocks, to asymmetric information and signal extraction problem. For example, Ehrmann and Smets (2003) show that the effects of a cost-push shock become more persistent when the private sector cannot distinguish between temporary cost-push shocks and permanent shocks to potential output. Similarly, Erceg and Levin (2003) explain the persistent effects of the Volcker disinflation period by the private sector's learning about whether the monetary policy-induced fall in inflation is permanent or temporary. The source of asymmetric information on behalf of the private agents can also be due to a lack of knowledge about the central bank's inflation target (Kozicki and Tinsley, 2003) or uncertainty about the central bank's preferences of inflation over real activity (see for example Cukierman and Meltzer, 1986; Tetlow and von zur Muehlen, 2001). Dossche and Everaert (2005) argue that if private agents have to extract information about the central bank's inflation target from a monetary policy rule, the signal-to-noise ratio of this policy rule determines the uncertainty faced by private agents in disentangling transitory and permanent policy shocks and therefore also the speed at which they recognize permanent policy shocks (see also Mankiw and Reis, 2002).

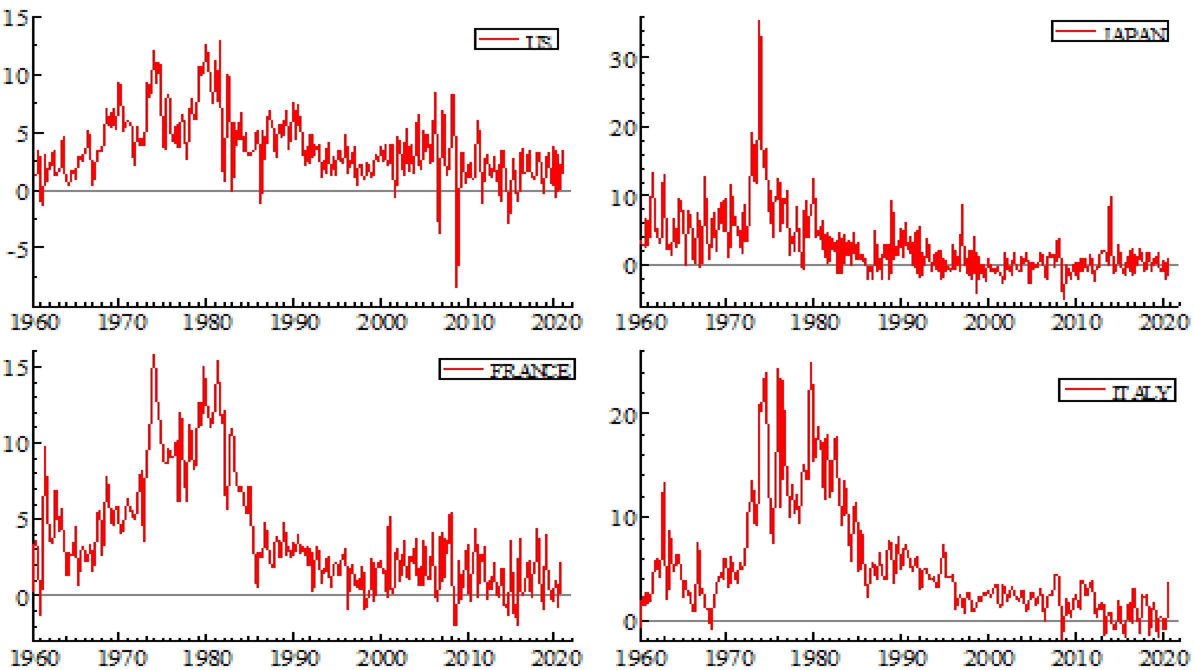
3 Empirical Work

3.1 Data

We apply the model discussed in Section 2 to the seasonally adjusted quarterly consumer price index (CPI) for the US, Japan, France and Italy over the period 1960Ql-2021Ql. Inflation is measured by quarterly difference of the $log(CPI)$ [i.e. $Y_t = 400\log(CPI_i/CPI_{t-1})$]. The data were taken from the Economic Research Federal Reserve Bank of St. Louis.

These countries are of interest because they adopted major monetary policy measures during the period under consideration. France and Italy's economies have undertaken several structural changes in the period under consideration due to the creation of the Economic and Monetary Union (EMU) with the introduction of a new currency and a new central bank responsible for preparing and implementing the single monetary policy. The Japanese economy suffered a prolonged stagnation in the 1990s followed by a major monetary easing policy. Moreover, the Bank of Japan has introduced various unconventional monetary policy tools since the launch of Abenomics in 2013, to achieve the price stability target of 2 percent inflation. The US suffered the "great inflation" period that started in the mid-60s and lasted for two decades in addition to an important deflation period after the subprime crisis, which caused a worldwide recession in 2005.

The logarithm of the CPI index is depicted in Fig. 1. The inflation series show a great deal of variability over time, with a sharp rise between the 60s and early 80s in France, Italy and the US, followed by a sharp decline in the 90s. In Japan, the inflation rate picked up during the first oil shock in the early 1970s, whereas it shows a more stable pattern starting from the early 1990s.



CPI in the US, Japan, France and Italy from 1960Q1-2021Q1.

3.2 Test for Structural Breaks

The first step prior to the estimation procedure is to identify possible points of parameter changes. With this target in mind, the Bai and Perron (2003) sequential test is used to identify possible breaks in the inflation series during the sample period under consideration.[2] Bai and Perron (2003) proposed an F-type test for l versus $l+1$ breaks, which we refer to as *sup* $F_t(l+1|l)$. The testing procedure allows for a specific to general modelling strategy for the determination of the number of breaks in each series. The test is applied to each segment containing the T_{i-1} to $T_i (i = 1, ..., l+1)$. In particular, the procedure involves using a sequence of $(l+1)$ tests, where the conclusion of a rejection in favour of a model with $(l+1)$ breaks if the overall minimal value of the sum of squared residuals is sufficiently smaller than the sum of the squared residuals from the l break model.

[2] Since the seminal paper by Perron (1989) a great deal of research has been directed to the detection and estimation of breaks, and forecasting in the presence of breaks, see for example Iacone et al. (2021) and the references therein.

Note that the sum of the squared residuals is calculated over all segments where an additional break is included and compared with the residuals from the l model. Therefore, the break date selected is the one associated with the overall minimum.

The results of the structural break test are reported in Table 1. The first column reports the null hypothesis of l breaks versus the alternative hypothesis of $l + 1$ breaks, the second column reports the calculated value of the statistics and the third column the critical value of the test.

Table 2. Bay and Perron test of $L + 1$ vs L sequentially determined breaks.

Null Hypothesis	U.S.	Japan	France	Italy
$\mathcal{H}_0$: 0 vs 1	67.46^{**}	88.03^{**}	180.78^{**}	100.46^{**}
$\mathcal{H}_0$: 1 vs 2	58.25^{**}	66.16^{**}	146.51^{**}	177.11^{**}
$\mathcal{H}_0$: 2 vs 3	26.61^{**}	10.09	33.84^{**}	169.33^{**}
$\mathcal{H}_0$: 3 vs 4	14.61^{**}	-	4.85	27.46^{**}
$\mathcal{H}_0$: 4 vs 5	0.000	-	-	0.00
Break dates	1969Q3	1981Q2	1973Q1	1972Q3
	1981Q4	1993Q3	1985Q2	1986Q1
	1991Q2	-	1996Q2	1996Q2
	2008Q3	-	-	2012Q1

Note: **) indicates significance at the 5% level.

Looking at the calculated values of the test it appears that the null hypothesis of zero versus one break is rejected in favour of the alternative hypothesis for all countries. The hypothesis of one break versus two breaks is rejected for Japan. However, the null hypothesis of two versus three breaks is not rejected, therefore we conclude that there are two structural breaks in the inflation series of Japan. According to the results in Table 1 we cannot reject the null hypothesis of four versus five breaks for Italy and the United States, whereas we reject the null hypothesis of two versus three breaks in favour of the alternative for France. Therefore, we conclude that there are four breaks in the inflation series of the US and Italy and three structural breaks in France.

Four breaks are found in the US, the first break occurred in 1969 when President Nixon took office. The other two breaks occurred around the Volcker monetary regime period when the FED used interest rates to create a nominal anchor in the form of an expected low, stable trend inflation. The last break occurred in 2008 in the wake of the financial crisis. After the stock market crashed in 2005 the country entered the Great Recession, which led to a period of deflation in 2009. Note that similar break dates were found in the related literature (see, for example, Caporin and Gupta, 2017).

As for Italy and France, the two countries experienced mostly synchronised breaks. The first break occurred in the early 70s during the oil shock crisis. The second break occurred in the mid-1980s with the launch of the Single Market Programme in Europe in 1985. From 1986 onwards, the Italian and the French Central Banks started to intervene more frequently in the market; therefore, monetary policy interventions may have caused the break observed in 1996 in these countries (see Bilke, 2004). Note that, in the literature, similar results were found by Corvoisier and Mojon (2004), who detected two breaks in France, in 1973 and 1985, whereas Benati (2008) and Gadzinski and Orlandi (2004) found a second break in mean of inflation at the beginning of the nineties. Finally, the last break in Italy occurred in the wake of the sovereign debt crisis in 2012 when the country incurred a deep recession. Coming to Japan in the 1990s, the country was hit by a deflation period after the economic bubble burst, therefore the break in 1992 might reflect that change in economic conditions.

Accordingly, below we estimate the model in Eq. (1)-(2), allowing for both inflation's intrinsic persistence and uncertainty's persistence to switch across breakpoints. This should enable us to determine whether changes in the structure of the conditional mean of inflation observed in these countries derive from changes in the estimated parameters in the conditional mean and/or the conditional variance equations. We use dummy variables that take the value zero in the period before each break and the value one after the break to capture these changes.

3.3 Estimation Results

As far as the results are concerned Table 3 reports the estimated parameters of the AR(1)- APGARCH(1,1)-ML model for each of the four countries and the relative misspecification tests. In particular, the top part of Table 3 reports the estimated parameters for the conditional mean, whereas the coefficients for the conditional variance are given in the bottom part. Note that in the preliminary model selection procedure for each of the inflation series a number of specifications were estimated, however, in Table 3 we only report the estimated coefficients for the best model specification. The model selection was undertaken according to the Akaike and Schwarz information criteria.

Table 3. Estimated AR(1) APGARCH(1,1)-ML with time varying parameters.

	US	Japan	France	Italy
Conditional Mean Equation				
φ	0.007^{***} (0.001)	-6.633^{***} (1.713)	2.257^{***} (0.139)	2.026^{***} (0.615)
ϕ_0	0.225^{***} (0.046)	-0.247^{***} (0.052)	0.638^{***} (0.060)	-0.418^{***} (0.076)
ϕ_1	–	–	0.183^{***} (0.054)	–
ϕ_2	–	–	-0.185^{***} (0.008)	–
ϕ_3	–	–	-0.040^{***} (0.009)	–
ς_0	0.472^{***} (0.039)	3.356^{***} (0.669)	-0.377^{***} (0.096)	0.166^{*} (0.089)
ς_1	1.530^{***} (0.281)	–	–	1.003^{***} (0.286)
ς_2	-0.707^{***} (0.273)	–	–	-0.651^{**} (0.268)
ς_3	-0.815^{***} (0.220)	–	–	-0.960^{***} (0.210)
ς_4	–	–	–	-0.354^{*} (0.180)
Conditional Variance Equation				
ω	0.0004 (0.000)	1.288^{***} (0.745)	0.545 (0.343)	0.025 (0.122)
α_0	0.135^{***} (0.051)	0.020^{**} (0.008)	0.178^{**} (0.086)	0.155^{*} (0.092)
β_0	0.753^{***} (0.107)	0.832^{***} (0.037)	0.546^{**} (0.228)	0.659^{***} (0.127)
β_1	–	-0.695^{***} (0.020)	–	-0.019^{**} (0.009)
β_2	–	-0.032^{**} (0.013)	–	-0.039^{*} (0.021)
β_3	–	–	–	0.051^{**} (0.026)
β_4	–	–	–	0.091^{**} (0.040)
ϱ	0.041^{*} (0.024)	0.167^{**} (0.070)	–	0.103^{***} (0.03)
Conditional variance persistence				
c_0	0.992	0.867	0.861	0.933
c_1	–	0.172	–	0.952
c_2	–	0.140	–	0.992
c_3	–	–	–	0.940
c_4	–	–	–	0.849
Q-Statistics (4)	5.234 [0.264]	2.165 [0.7054]	0.962 [0.915]	2.734 [0.633]
Log Likelihood	−934.79	−583.91	−932.20	−491.02

Note: *, **, *** indicate statistical significance at 10%, 5% and 1%, respectively. The numbers in parentheses are standard errors. The numbers in brackets are p-values.

From Table 3, it is clear that breaks in the parameters are an important feature of inflation dynamics for all the series under consideration. However, none of the models with breaks in the parameter α of the conditional variance equation outperformed alternative specifications according to the information criteria. This implies that breaks in the intrinsic persistence of the conditional mean equation and/or the intrinsic conditional variance better fit the series under consideration.

Looking now at the estimated parameters, according to the results in the top part of Table 3 the US and Italy share in common three and four breaks in the in-mean parameter, respectively. In contrast, the inflation process for France is better approximated using time-varying parameters for the autoregressive coefficient. Finally, the best model for the inflation dynamic in Japan has time varying parameters in the conditional variance equation but not in the conditional mean equation. Note that all the models were estimated with $\delta = 1$.

The inflation process for the US is well approximated by a first-order autoregression with low intrinsic persistence ($\phi_0 = 0.225$). From the top part of Table 3 it appears that the inflation uncertainty imposed a moderate upward pressure on the inflation level in 1960Q1-1969Q3, but the impact sharply increased in 1969Q4-1981Q4 when the estimated coefficient rises from $\varsigma_0 = 0.472$ to $\varsigma_1 = 1.530$. During this period an expansion of social programs was undertaken by the US administration in the aftermath of a contraction period when unemployment and inflation reached high levels. However, from 1982 onward, the magnitude of the in-mean parameter decreases considerably, since in 1982Q1-1991Q2, the estimated combined coefficient reduces to $\varsigma_2 = |0.707|$ and remains approximately stable in 1991Q3-2021Q1.[3] Looking at the estimated sign, it is positive before the 90s, and negative afterword. This result suggests that the impact of inflation uncertainty on inflation level is in line with the Cukierman and Meltzer (1986) hypothesis of a positive correlation between inflation and its uncertainty. However, starting from the so called Volcker period in the 80s, the sign of the correlation reversed, thus suggesting that higher inflation variability lowers inflation. These results are in line with Holland (1995), who postulated that higher inflation uncertainty leads to lower average inflation.

Coming to Italy, from Table 3 it appears that the estimated in-mean coefficient sharply increased between 1972 and 1986, but it declined after the launch of the Single Market Programme in Europe in 1985, only to remain relatively low after 2012 when $\varsigma_4 = -0.354$.

Looking at the results for France, the best estimated model suggests that inflation persistence became less severe after the introduction of the single market, given that the estimated sign of the autoregressive parameter ϕ is negative after 1985 and it decreases in magnitude from $\phi_1 = 0.638$ in 1960Q1-1973Q1 to $\phi_2 = |0.183|$ in the period 1985Q2-1996Q2, and finally $\phi_3 = |0.040|$ in 1996Q3-2021Q1.

Considering now the estimated parameters of the conditional variance, it is interesting to note that in Japan the "intrinsic persistence" of the variance rapidly decreased in the 90s as the estimated parameter decreased from $\beta_0 = 0.832$ in 1960Q1-1981Q2 to $\beta_1 = |0.695|$, only to remain extremely low after 1993Q3. In the 1990s, the Japanese economy suffered a prolonged recession that followed the collapse of the economic bubble in the 1980s. This stretch of economic stagnation finally ended in 2002, after more than 10 years.

Looking at the specication test the reported values of the Ljung-Box Q statistic does not reject the null hypothesis that there is no autocorrelation up to the forth order, thus indicating the absence of serial

[3] Note that the estimated parameter for the break in 2008Q3 was not significant. Therefore, the related variable was removed by the estimated model.

correlation for all the estimated models.

We now employ the estimation results to compute the persistence measure presented in Section 2. Table 4 presents the time varying persistence for the four countries under consideration. From Table 4, it appears that when accounting for the transmission of memory from the conditional variance to the conditional mean the overall picture changes since the calculated value of π_t is relatively high although we observe a moderate tendency to decrease over time. In particular, looking at the US there is evidence that persistence has been relatively high over the 60s, but it shows a moderate tendency to reduce over time. This result is in agreement with Pivetta and Reis (2007), where it was found that inflation persistence in the US was high over time over the period 1965-2001.

Coming to the European Countries, in Italy, there is evidence that inflation persistence moderately declined from the 1960s to the end of the 20th century. It further decreased in the last twenty years when the European Central Bank adopted responsibility for the monetary policy of the Euro-area countries. Similarly, the inflation persistence was relatively high in France. However, from Table 4, it appears that Europe's Economic and Monetary Union (EMU) had little impact on the dynamic of the inflation persistence. These results are in line with Angeloni et al. (2006). The authors investigated whether the EMU altered the behaviour of retail price setting and inflation dynamics and found no evidence that persistence changed with the introduction of the EMU.

Considering now Japan, evidence points, once again, against the notion of inflation as a uniformly highly persistent process. From Table 4, it is clear that persistence has significantly declined starting from the 1980s. This result may be explained by the fact that the economy entered a period of low growth and deflation in this country after the burst of the asset bubble in the early 1990s.

Table 4. Inflation persistence' for each of the four countries under consideration.

US		Japan		France		Italy	
Date	$\pi(t)$	Date	$\pi(t)$	Date	$\pi(t)$	Date	$\pi(t)$
1960Q1-1969Q3	0.975	1960Q1-1981Q2	0.957	1960Q1-1973Q1	0.949	1960Q1-1972Q3	0.949
1969Q4-1981Q4	0.951	1981Q3-1993Q3	0.608	1973Q2-1985Q2	0.924	1972Q4-1986Q1	0.959
1982Q1-1991Q2	0.941	1993Q4-2021Q1	0.488	1985Q3-1996Q2	0.950	1986Q2-1996Q2	0.912
1991Q3-2021Q1	0.907	–	–	1996Q4-2021Q1	0.959	1992Q3-1996Q2	0.905
–	–	–	–	–	–	1996Q3-2021Q1	0.678

Note: The inflation persistence has been calculated according Eq. (10) using the estimated parameters in Table 3.

4 Testing for Stationarity: A Monte Carlo Experiment

Researchers make use of unit root tests as the first test of persistence since if the inflation series inflation contains a unit root, its persistence is large (infinite), and its variance is unbounded. A frequent criticism of unit root tests concerns the poor power and size properties that many such tests exhibit. Standard unit root tests are based on the assumption that some type of heteroscedasticity is present, but ignore the possibility that the volatility has a direct impact on the level. The model in Eq. (1) and (2) assumes that the autocovariance function of y_t is increasing in the parameters c and ς (see Conrad and Karanasos, 2015). This implies that even if $\phi = 0$ (i.e. there is no intrinsic persistence) the inflation process exhibits autocorrelation and the magnitude of the correlation is increasing in the in-mean parameter. In this respect, the estimation results in Table 3 may explain why the literature on inflation persistence provides mixed evidence in support of the existence of a unit root in inflation series. Accordingly, the question we are trying to answer in this section is the following: Assume that the inflation level is generated by the data generating process (DGP) in Eq. (1) and (2), but the empirical investigator assumes that the process follows an autoregressive process. How often will this researcher reject a unit root in the process when using unit root tests?

To answer this question, we undertake a Monte Carlo simulation exercise to examine the performance of commonly used unit root tests in cases where the unknown DGP of the inflation process follows an AR(1)-APGARCH-M model. The two unit root tests considered are the Dickey-Fuller test (DF) proposed by Dickey and Fuller (1979) and the M test proposed by Perron and Ng (1996). As far as the estimation of the autoregressive parameter ϕ is concerned both the ordinary least squared method (OLS) and the generalized least squared method (GLS) suggested by Elliott *et al.* (1996) are considered. This gives us two DF statistics, which we define as DF_{OLS} or DF_{GLS} depending on the estimation method used for ϕ. Likewise, the M tests are defined as M_{OLS} and M_{GLS} respectively.

The DGP used for the Monte Carlo simulation experiment was the model in Eq. (1) and (2) with

$$\varphi(t) = \phi(t) = 1 \text{ for all } t,\ \delta = 1,\ \omega = 1 - \alpha - \beta,\ \alpha = 0.1,\ \beta = 0.70,\ \gamma = 0,\ \varrho = 0, \tag{11}$$

and there are two abrupt breaks in the time varying in-mean coefficient, $\varsigma(t)$, at times $t - k_1$ and $t - k_2$. In particular, $\varsigma(\tau) = \varsigma_1$ for $\tau < t - k_2$ and $\tau > t - k_1$, whereas $\varsigma(\tau) = \varsigma_2 = \varsigma_1 + \Delta_\varsigma$ for $t - k_2 \leq \tau \leq t - k_1$. The magnitude of the break is denoted by Δ_ς and the length of the break by $\Delta_k = k_2 - k_1$. Therefore, time variation is caused only by the in-mean coefficient. We also set the sample size k equal to $1,000$. Finally, $\{e_t\}$ are $i.i.d \sim N(0,1)$ random variables.

The Monte Carlo simulation experiment design is targeted at investigating the effect of the in-mean breaks on the empirical sizes of the test statistics under consideration. However, as the magnitude of the in-mean parameter itself is likely to affect the performance of the test statistics we investigate this latter issue before considering the former. Accordingly, the Monte Carlo experiment is aimed at investigating the effects on the empirical sizes of *i*) the magnitude of the in-mean parameter, *ii*) the magnitude of the break, Δ_ς, and *iii*) the timing (k_1, k_2) and the length or duration (Δ_k) of the breaks as a fraction of the sample size, k. To address point *i)* a set of simulation experiments was undertaken with the DGP in eqs. (1), (2) and (11) with increasing magnitude of the in-mean parameter, namely $\varsigma_1 \in \{0.1, 0.3, 0.9\}$. Similarly, to investigate point *ii*) simulation experiments were undertaken with $\Delta_\varsigma \in \{0.07, 0.25, 0.50\}$ with the case of $\Delta_\varsigma = 0.00$ set as benchmark. Finally, to tackle point *iii*) in the experiment design we considered the above DGP with $k_1/k = (k - k_2)/k \in \{0.100, 0.333, 0.450\}$, that is $k_1 = (k - k_2) \in \{100, 333, 450\}$. In other words, we consider three values for the length of the in-mean break: $\Delta_k/k = (k_2 - k_1)/k \in \{0.80, 0.333, 0.10\}$ or $\Delta_k \in \{800, 333, 100\}$.

Note that all experiments were performed over $10,000$ Monte Carlo replications using, as noted earlier, a sample size $k = 1,000$, with a further 50 observations created and discarded in order to avoid the influence of the initial values. The sequence $\{e_t\}$ was generated using pseudo $i.i.d{\sim}\ N(0,1)$ random numbers from the RNDNS procedure in GAUSS with the value of y_0 set as a $N(0,1)$ random number.

Table 5 reports the results for the empirical sizes of the inference procedures under consideration for the 5% nominal significance level. The top panel reports the empirical sizes resulting from the simulation experiment for the DGP with $\Delta_k = 800$, whereas the results for $\Delta_k = 333$ and $\Delta_k = 100$ are given in the middle and bottom panel, respectively.

Table 5. Empirical sizes of unit root tests: the case of two unknown breaks in the in-mean parameter.

		DF_{OLS}	DF_{GLS}	M_{OLS}	M_{GLS}
		$\Delta_k = 800$ or $\Delta_k/k = 0.80$			
$\varsigma_1 = 0.1$	$\Delta_\varsigma = 0.00$	0.049	0.054	0.054	0.054
	$\Delta_\varsigma = 0.07$	0.048	0.042	0.046	0.040
	$\Delta_\varsigma = 0.25$	0.048	0.037	0.037	0.037
	$\Delta_\varsigma = 0.50$	0.017	0.011	0.012	0.011
$\varsigma_1 = 0.3$	$\Delta_\varsigma = 0.00$	0.049	0.040	0.042	0.040
	$\Delta_\varsigma = 0.07$	0.045	0.028	0.030	0.028
	$\Delta_\varsigma = 0.25$	0.029	0.014	0.014	0.013
	$\Delta_\varsigma = 0.50$	0.012	0.007	0.005	0.006
$\varsigma_1 = 0.9$	$\Delta_\varsigma = 0.00$	0.015	0.001	0.005	0.001
	$\Delta_\varsigma = 0.07$	0.013	0.001	0.001	0.001
	$\Delta_\varsigma = 0.25$	0.016	0.000	0.000	0.000
	$\Delta_\varsigma = 0.50$	0.008	0.000	0.000	0.000
		$\Delta_k = 333$ or $\Delta_k/k = 0.333$			
$\varsigma_1 = 0.1$	$\Delta_\varsigma = 0.07$	0.047	0.045	0.046	0.045
	$\Delta_\varsigma = 0.25$	0.049	0.042	0.043	0.041
	$\Delta_\varsigma = 0.50$	0.020	0.027	0.022	0.025
$\varsigma_1 = 0.3$	$\Delta_\varsigma = 0.07$	0.042	0.030	0.032	0.030
	$\Delta_\varsigma = 0.25$	0.037	0.025	0.024	0.025
	$\Delta_\varsigma = 0.50$	0.013	0.010	0.009	0.010
$\varsigma_1 = 0.9$	$\Delta_\varsigma = 0.07$	0.013	0.001	0.002	0.001
	$\Delta_\varsigma = 0.25$	0.009	0.000	0.000	0.000
	$\Delta_\varsigma = 0.50$	0.003	0.000	0.000	0.000
		$\Delta_k = 100$ or $\Delta_k/k = 0.10$			
$\varsigma_1 = 0.1$	$\Delta_\varsigma = 0.07$	0.049	0.053	0.043	0.053
	$\Delta_\varsigma = 0.25$	0.052	0.038	0.044	0.037
	$\Delta_\varsigma = 0.50$	0.037	0.045	0.046	0.045
$\varsigma_1 = 0.3$	$\Delta_\varsigma = 0.07$	0.048	0.038	0.031	0.038
	$\Delta_\varsigma = 0.25$	0.050	0.036	0.031	0.035
	$\Delta_\varsigma = 0.50$	0.022	0.023	0.019	0.023
$\varsigma_1 = 0.9$	$\Delta_\varsigma = 0.07$	0.013	0.001	0.002	0.001
	$\Delta_\varsigma = 0.25$	0.013	0.000	0.000	0.000
	$\Delta_\varsigma = 0.50$	0.008	0.001	0.001	0.001

Note: The DGP is $y_t = 1 + y_{t-1} + \varsigma(t)\sigma_t + e_t\sigma_t$ and $\sigma_t = 0.2 + 0.1\,|e_{t-1}\sigma_{t-1}| + 0.7\sigma_{t-1}$, where $\varsigma(\tau) = \varsigma_1$ if $\tau > t - k_1$ or $\tau < t - k_2$, and $\varsigma(\tau) = \varsigma_2 = \varsigma_1 + \Delta_\varsigma$ otherwise with $\varsigma_1 \in \{0.1, 0.3, 0.9\}$, $\Delta_\varsigma \in \{0.07, 0.25, 0.50\}$, $k = 1,000$, $k_1 = (k - k_2) \in \{100, 333, 450\}$ or $\Delta_k \in \{800, 333, 100\}$.

Looking at the results in Table 5, we first notice that all inference procedures appear to be robust to small values of the in-mean parameter ($\varsigma_1 = 0.1$) and of the breaks ($\Delta_\varsigma = 0.07$). However, the magnitude of the in-mean parameter appears to have a significant effect on the size distortion of all test statistics since, even when $\Delta_{\varsigma} = 0.00$, for $\varsigma_1 = 0.9$ all the test statistics are severely undersized. Similarly, both the magnitude and the location of the breaks affect the size properties of the inference procedures under consideration as from the top panel of Table 5 it is clear that the worst case scenario appears to be when $\Delta_k = 800$ and $\Delta_{\varsigma} \geq 0.25$. In this case, the break occurs very early and the stochastic process stays in the second regime for 80% of the time period, only to go back to the first regime for the last 100 observations.[4]

Looking now at the performance of the individual tests, it appears that the *OLS* based tests are more robust to regime shifts in the in-mean parameter than the *GLS* based tests, as both DF_{OLS} and M_{OLS} enjoy smaller size distortion than their *GLS* based counterparts.

4.1 Empirical Power

The empirical sizes of the unit root tests presented in Table 5 are constructed to generate a test with asymptotic size of 5% under the null hypothesis of a unit root. We now focus on examining the power of the inference procedures to reject the null hypothesis of $\phi(t) = 1$ for all t when in fact the process is second-order, that is $\phi(t) = \phi$ with $|\phi| < 1$ for all t.

As for the size, the Monte Carlo experiment design is meant to investigate the effects for points *i*) - *iii*) above. With this target in mind, the asymptotic local power functions for the 5% nominal level test have been calculated. To model the sequence of stationary alternatives near the null hypothesis of unit root, we consider the aforementioned *DGP* but now with $\phi(t) = 1 - \frac{l}{k}$ for all t (instead of $\phi(t) = 1$) in eq. (1) where $l = 30, 29, ..., 1, 0$ controlling the size of the departure from a unit root.

To investigate the issue in point *i*) simulation experiments were undertaken setting different values of the in-mean parameter under the alternative hypothesis. The simulation results are summarized in Figure 2, where the asymptotic local power curves are plotted for the *DGP* when the magnitude of the parameter is increased from the modest value of $\varsigma_1 = 0.1$ to a relatively large value $\varsigma_1 = 0.9$, with the break parameter fixed at $\Delta_{\varsigma} = 0$. In the x-axis the value taken by l is reported, whereas in the y-axis the empirical rejection frequencies are reported. Looking at the plot of the asymptotic power curves for the tests under consideration from Figure 1 it appears that all test statistics are sensitive to the magnitude of the in-mean parameter. However, it is clear that DF_{OLS} and M_{OLS} are less sensitive to the magnitude of ς than the *GLS* based counterparts.

Coming to target point *ii*), in Figure 3 we report the results of simulation experiments obtained by fixing the in-mean parameter at 0.9 and $\Delta_k = 800$, then comparing the resulting power curves of the test statistics when $\Delta_{\varsigma} = 0$ and $\Delta_{\varsigma} = 0.5$. Interestingly enough, the DF_{OLS} procedure appears to be the most robust to the regime shift of the in-mean parameter. By contrast both *GLS* based statistics are

[4] We also find (results not reported) that in the presence of asymmetries the size distortion of the unit root tests is stronger.

severely affected by the magnitude of the break.

Finally, we consider the issue of the timing and duration of the in-mean regime shift as stated in target point *iii*). In this case the simulation experiment was undertaken with $\Delta_k \in \{800, 100\}$ and Δ_ς fixed at the smallest value 0.07. Figure 4 plots the asymptotic local power function for DF_{OLS}, DF_{GLS}, M_{OLS} and M_{GLS} respectively. From the results in Figure 3 it appears that the empirical power of all inference procedures is less affected by the timing and the duration of the regime shift than the size reported in Table 5. Note that in the interest of brevity not all the values of the parameter space considered in Table 5 have been reported, but results are available upon request.

5 Discussion and Monetary Policy Implications

What do we learn from this application? First, from the results in Table 3 and Table 4, it is clear that when evaluating the inflation dynamic, any suggested measure of persistence needs to take into account the complex data generating process of the inflation series. Theoretical models indicate that the inflation data generating process should incorporate a number of distinct components, each exhibiting its own level of persistence (see, for example, Dossche and Everaert, 2005). However, a common practice in empirical works is to make use of autoregressive models and draw inference on the level of persistence from the analysis of the estimated autoregressive coefficients (see, for example, Robalo, 2004; Pivetta and Reis, 2007; Fuhrer, 2010). The main point highlighted in this paper is that such measures of persistence may blur the picture of the inflation dynamic by classifying "expectations-based persistence" as "intrinsic persistence". This fact has far-reaching consequences for monetary policy since "intrinsic persistence" is structural and difficult to eradicate without incurring a recession, whereas "expectations-based persistence" can be managed by a transparent monetary policy. In this respect, empirical evidence suggests that when inflation expectations are well-anchored, the intrinsic component of the inflation formation mechanism becomes less important and shocks to inflation will be less persistent. For example, the introduction of inflation targeting adopted by many countries aiming at maintaining price stability after the late 1980s resulted in a reduction of inflation volatility with respect to volatility witnessed in the 1970s and the early 1980s (see, for example, Samarina, 2014).

Second, from the results in Table 3, it appears the estimated coefficients of the "intrinsic persistence" are generally speaking rather small in magnitude. These findings align with a strand of literature that uses structural models to explain inflation persistence. For example, Benati (2008) considers the inflation series in the United Kingdom and the Euro area and finds that the degree of intrinsic inflation persistence as captured by the coefficient of lagged inflation in a hybrid Phillips curve drops significantly towards zero once a credible new monetary regime is in place. Other influential studies that make use of structural models are Angeloni et al. (2003), Benigno and Lopez-Salido (2002), Gali et al. (2001), Zhang et al. (2008), Jondeau and Le Bihan (2005), McAdam and Willman (2004), Rumler (2005). These papers estimate structural time-series models in the New Keynesian Phillips Curve framework, where the inflation rate depends on the deviation of the actual from the desired mark-up and an exogenous mark-up shock. Most of these studies use a small stylised model of the economy consisting of an IS relation in addition to the Phillips curve. In this framework, the IS curve links the current output gap to its own lagged and future expected value, the real interest rate, and a demand shock. In addition, an equation capturing the behaviour of the policymaker is often added, and it is assumed the policy maker acts either according to the Taylor rule or in an "optimal" fashion, depending on the structure of the economy. In line with the results in Table 3, these structural time series models find that estimates of the backwards-looking parameters are relatively low, whereas expectations-based and extrinsic are major sources of persistence.

In sharp contrast to the results of structural models where the various components driving inflation are explicitly modelled, empirical works that rely on reduced form estimates of persistence report high intrinsic persistence levels in the countries under consideration. In most of these studies, inflation is found to be close to a random walk. See, for example, Altissimo et al. (2004); Batini (2001); Gadzinski and Orlandi (2004); O'Reilly and Whelan (2004); Marques (2004), among others. These results suggest that policy makers pursuing price stability should entail a more aggressive policy response to economic shocks with respect to the stance that estimation results from structural models would indicate. In this respect, the findings of this paper seem to reconcile different results found by structural models and reduced-form estimation since, from Table 4, it is clear using a persistence measure obtained by reparametrizing the AR(1)-APGARCH-(1,1)-ML model into the ARMA(2,1) form, we obtain results in line with reduce-form persistence measure literature. However, the results in Table 3 suggest that when the distinct components of the inflation process are taken into consideration, most of the persistence is due to the "expectations-based" component (as proxied by uncertainty) rather than the "intrinsic persistence" component. These results reiterate the main point highlighted in this paper that estimates of high post-WW II inflation persistence obtained from autoregressive models are hard to interpret and have the potential of blurring the lesson that stability-oriented policy makers can learn from them. Third, from the estimation results in Table 4, it is clear that any meaningful measure of persistence has to allow for structural breaks. Failure to account for breaks may yield spuriously high estimates of the degree of persistence. In the literature, the issue of structural shifts in the inflation process is crucially important. It is, therefore, not surprising that great efforts have been devoted to account for shifts in the central bank's target. For example, O'Reilly and Whelan (2004) and Pivetta and Reis (2007) use rolling regressions to allow for breaks in the conditional mean of inflation over different periods. Levin and Piger (2004), Gadzinski and Orlandi (2004) and Bilke (2004) estimate an autoregressive model allowing for discrete breaks in the mean of the inflation process. Cogley and Sargent (2001) estimate time-varying autoregressive coefficients conditional on a time-varying mean. However, most of these works allow for breaks in the conditional mean only, neglecting the role of breaks in the conditional variance. The implication of the AR(1)-APGARCH-ML model is that a higher level of uncertainty increases inflation persistence. In this model, a large increase in the persistence of the conditional variance induces the inflation process to behave like a non-stationary process. This will be the case if there is high persistence in the conditional variance and the transmission of memory from the conditional variance to the conditional mean is sufficiently strong. In this respect, the Monte Carlo experiment results reinforce the findings of Conrad and Karanasos (2015), who show that in GARCH-in mean models, the largest root of the autoregressive part is closely linked to the persistence of the conditional variance of the process. A similar argument is put forward by Rahbek and Nielsen (2014), who consider a multivariate model in which lagged levels enter both the conditional mean and the conditional variance and show that the process can be strictly stationary and ergodic although the individual series have unit roots.

Finally, the fact that the impact of inflation uncertainty changes over time has important policy implications since, from Table 3, it is clear that the estimated parameters for the in-mean coefficients have become smaller in the last thirty years. This period corresponds to the active inflation targeting policy undertaken by the central banks in these countries. These findings suggest that in a stable inflation regime environment, where the objectives of policy makers are clear and where the public perception about this inflation objective is well anchored, inflation persistence may be reduced. In other words, by conducting monetary policy such that inflation expectations of economic agents are well anchored, policy makers can make inflation expectations less dependent on backwards-looking inflation, thus ensuring that the actual inflation rate is not too far from their medium-term objective for inflation (see for example Gerlach and Tillmann, 2012).

6 Conclusion

In recent years economists have placed significant increasing emphasis on investigating persistence and structural shifts in the dynamics of the inflation process. A number of detailed and rigorous empirical studies regarding changes in inflation persistence have, however, reached diverging conclusions. Several studies find evidence of little or no change of inflation persistence over the past four decades, whereas others conclude that there has been a pronounced decline over the same period.

In this paper we have attempted to reconcile different strands of the literature by showing that seemingly conflicting results regarding changes in inflation persistence actually constitute two sides of the same problem. Our analysis relies on the estimation of an AR(1)-APGARCH-ML model that can be used as a base for computing a persistence measure that is able to account for impact of uncertainty on inflation persistence. By allowing the autoregressive coefficient of the conditional variance to change over time we can investigate the dynamic properties of inflation persistence. More importantly, analysing the transmission of memory from the conditional variance to the conditional mean we can investigate the impact on the dynamic properties of the inflation process.

Our empirical investigation shows that if the estimated model is misspecified with respect to the data generating process, commonly used test statistics to detect persistence would deliver spurious results.

In particular, using Monte Carlo simulation experiments we have shown that if the in-mean mechanism (together with the possible presence of breaks in the in-mean parameter) is ignored, conventional unit root tests might falsely indicate inflation as being a nonstationary rather than a stationary process. The obvious consequence is that commonly used inference procedures would suggest the modelling of inflation processes in their first differences rather than in their levels.

Our paper adds to the literature that has challenged the empirical relevance of the Lucas critique on reduced-form models. In this respect, from the empirical point of view our measure of persistence constitutes an important breakthrough in the literature since we allow for feedback from volatility (inflation uncertainty) to the level of the process (inflation).

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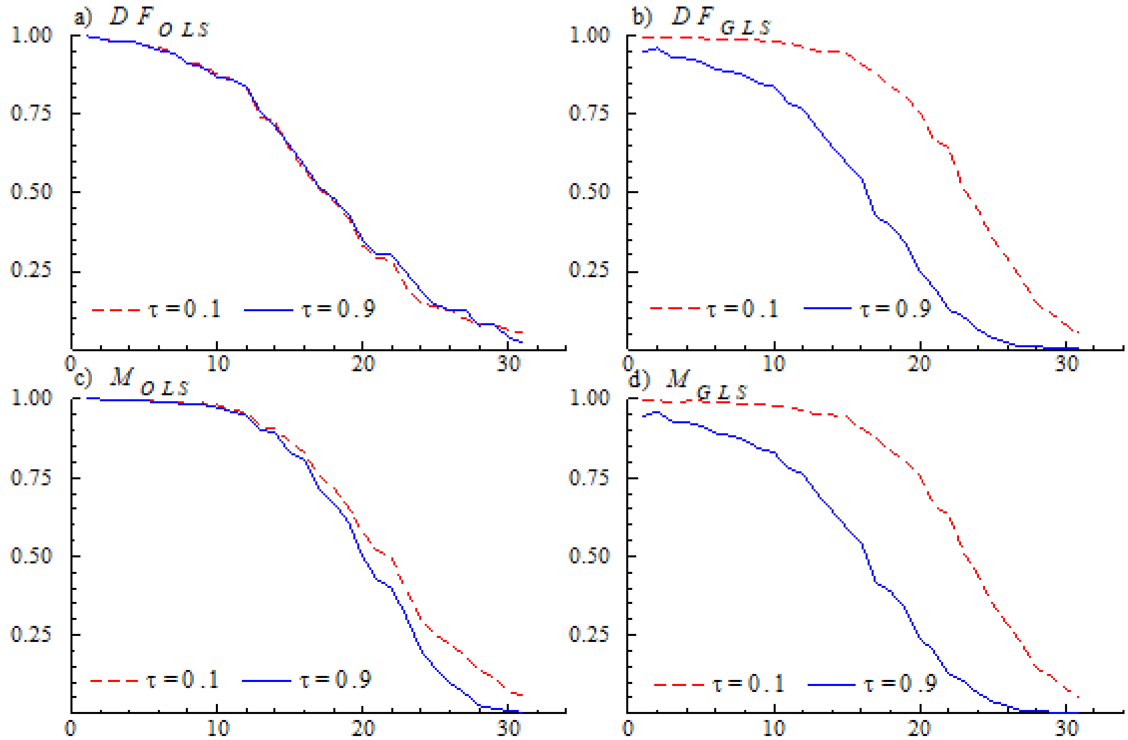

Figure 2. Power of DF and M tests. The DGP is $y_t = 1 + y_{t-1} + \varsigma\sigma_t + \varepsilon_t$ and $\sigma_t = 0.2 + 0.1\,|\varepsilon_{t-1}\sigma_{t-1}| + 0.7\sigma_{t-1}$.

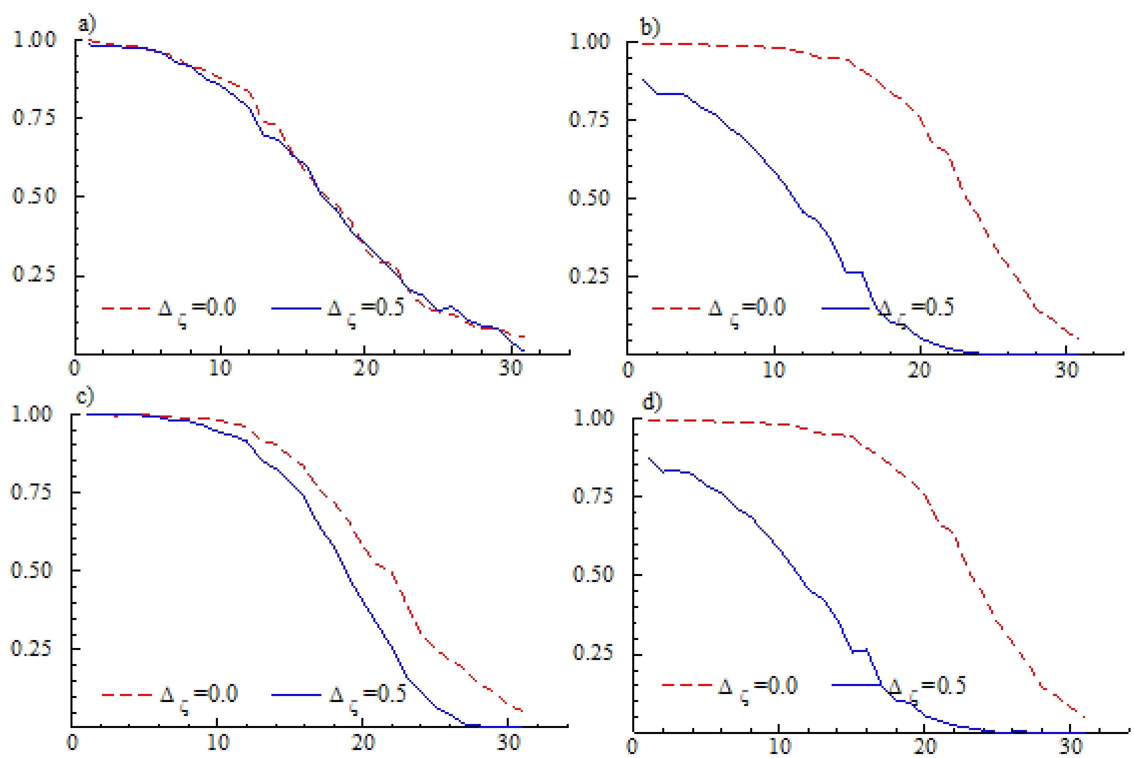

Figure 3: Power of DF and M tests. The DGP is $y_t = 1 + y_{t-1} + \varsigma(t)\,\sigma_t + \varepsilon_t$ and $\sigma_t = 0.2 + 0.1\,|\varepsilon_{t-1}\sigma_{t-1}| + 0.7\sigma_{t-1}$, where $\varsigma(\tau) = \varsigma_1$ if $\tau > t - k_1$ or $\tau < t - k_2$, and $\varsigma(\tau) = \varsigma_2 = \varsigma_1 + \Delta_\varsigma$ otherwise with $\varsigma_1 = 0.9$, $k = 1{,}000$, $k_1 = (k - k_2) = 100$ or $\Delta_k = 800$.

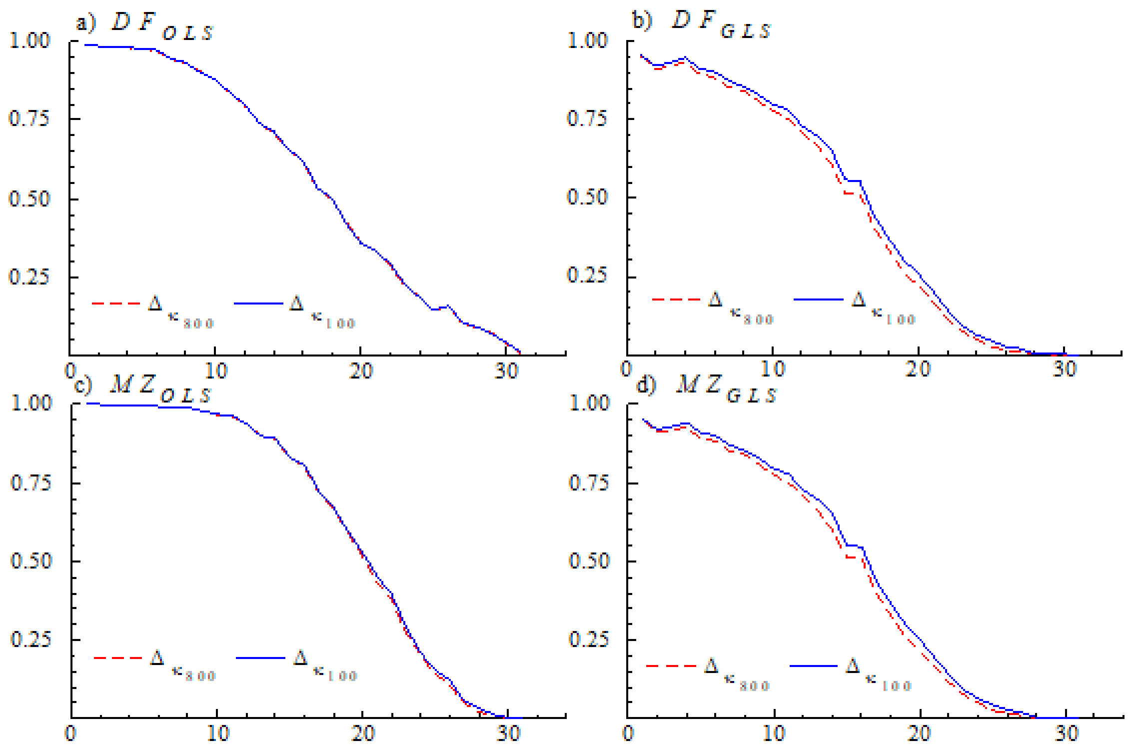

Figure 4. Power of DF and M tests. The DGP is $y_t = 1 + y_{t-1} + \varsigma(t)\,\sigma_t + \varepsilon_t$ and $\sigma_t = 0.2 + 0.1\,|\varepsilon_{t-1}\sigma_{t-1}| + 0.7\sigma_{t-1}$, where $\varsigma(\tau) = \varsigma_1$ if $\tau > t - k_1$ or $\tau < t - k_2$, and $\varsigma(\tau) = \varsigma_2 = \varsigma_1 + \Delta_\varsigma$ otherwise with $\varsigma_1 = 0.0$, $\Delta_\varsigma = 0.07$, $k = 1{,}000$, $k_1 = (k - k_2) \in \{100, 450\}$ or $\Delta_k \in \{800, 100\}$.