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THE ROLE OF PRECIOUS METALS IN PORTFOLIO DIVERSIFICATION DURING THE COVID₁₉ PANDEMIC: A WAVELET-BASED QUANTILE APPROACH

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The Role of Precious Metals in Portfolio Diversification During the Covid19 Pandemic: A Wavelet-Based Quantile Approach

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Abstract

In this study the relation between stock markets and precious metals during first wave of Covid-19 pandemic are investigated. We use a wavelet-based quantile procedure to investigate correlation between major stock markets of emerging countries (BRIC) and the United States. Our procedure reveals that precious metals offer market diversification opportunities during the period under consideration. In particular, it is found the gold, silver, platinum, and palladium, all serve as safe-haven assets during periods of market distress across short, medium and long investment horizons.

Key words: Precious metals; Stock markets, Quantile correlation, Wavelet Analysis.

JEL classification: G1, C14.

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1. Introduction

In the financial literature the correlation across market returns plays a crucial role in portfolio diversification. Markowitz (1952) suggests that investors should assemble an asset portfolio that maximised expected returns for a given level of risk. According to Markowitz model risk averse investors seek to minimise idiosyncratic risk by holding assets that are not perfectly positively correlated (for a recent review see for example Koumou, 2020). Since the seminal paper by Markowitz (1952) diversification as portfolio strategy has become one of the major components of investment decision-making under uncertainty. Portfolio diversification strategies are crucial not only to avoid country specific systemic risk, but also to dampen losses that investors may face due to price fluctuations in periods of market distress (see for example Levy, 1978; Levy and Roll, 2010).

This paper adds to the growing literature of portfolio diversification in periods of financial distress. We are particularly interested in investigating the role of precious metals in portfolio diversification during the early period of Covid-19 pandemic. Empirical studies have shown that investors seek to increase the proportion of low risk assets during period of market turmoil (see for example Joy, 2011; Daskalaki and Skiadopoulos, 2011; Arouri *et al.*, 2015).

The abundance of empirical evidence has prompted economists to seek explanations for the observed behaviour. For example, according to “flight to safety” theory, risk averse investment managers during periods of financial turmoil seek to sell assets perceived as risky to purchase safer assets. In the literature theoretical models that generate flight to safety behaviour are proposed. For example, according to the theoretical model in Vayanos (2004) risk averse investment managers during periods of financial turmoil require higher risk premiums which in turn drives down risky asset prices inducing investors to sell assets perceived as risky and purchase instead safe assets. In Caballero and Krishnamurthy (2008) increased uncertainty during period of financial turmoil leads agents to sell risky financial claims in favour of safe claims. Brunnermeier and Pedersen (2009) provide a model in which when funding liquidity is tight, traders become reluctant to take “capital intensive” positions in high-margin securities. The theoretical model explains the flight to quality behaviour triggered by a sharp drop in liquidity provision for the high margin, more volatile assets. In the literature representative agent models have also been used to explain flight to quality behaviour. In these models a “flight to safety” is typically defined as the joint occurrence of exogenous shocks leading to economic uncertainty with lower stock prices (induced by a cash flow or risk premium effect) and low real rates (through a precautionary savings effect) (Baele *et al.* 2020) (see also Beber *et al.*, 2009; Bekaert *et al.*, 2009).

Against this background, the uncertainty brought about by the global spread of Covid-19 has heightened market risk aversion in ways not seen since the global financial crisis (see OECD 2020 report). In this respect, a growing body of literature has found that the Covid-19 pandemic had an important effect on financial markets. For example, Baker *et al.* (2020) argue that in the U.S. stock markets volatility levels in the first quarter of 2020 surpassed those last seen in October 1987 and December 2008 and, before that, in late 1929 and the early 1930s (see also Al-Awadhi *et al.*, 2020; Alfaro *et al.*, 2020; Zhang *et al.*, 2020). For centuries precious metals such as gold and silver have been perceived as safe investment assets during period of bearish financial markets. However, the question has gained momentum during the Covid-19 pandemic since other types of low risk assets, such as sovereign bonds for example, that have been traditionally used as to balance portfolios in period of financial turmoil have been traded with negative rates. At the same time central banks have implemented expansive monetary policy measures keeping low interest rates to support the economy. The low interest rate environment

has reduced the opportunity cost of precious metals with respect of other forms of investments. In addition, fiscal support to the economy has raised concerns of a long-term run up of inflation caused by a widespread surge in government debt.

Against this background, the questions we ask in this paper are: Do precious metals provide valuable diversifying opportunities for equity portfolios during periods of financial turmoil? Consistent with modern portfolio theory the issue involves answering the question: Are the returns of precious metals negatively correlated to stock returns? Also, how does this correlation change in relation to stock market shocks? From the operational point of view, the question translates to: Does the sensitivity of precious metals to variation of stock market returns changes over their lower, upper, median quantiles of the precious metal distribution?

Finally, it is well known that different types of investors have different time horizons in their investing strategies. For example, day traders that seek to minimise the risk of their portfolios may have very different time horizons from pension fund managers that typically must ensure that eligible retirees receive the benefit they were promised. On the other side, hedge fund managers tend to invest in assets that can provide them good returns on investments within short-time. For this reason, they prefer liquid assets that allow them to shift portfolio allocation quickly. In this respect, evaluating the precious metal properties as potential portfolio stabilizer requires to investigate a broad spectrum of different time horizons to reflect the investors' heterogeneity. Accordingly, an additional issue we tackle in this paper is: How does the correlation between precious metals and stock markets changes over time and across investment horizons?

This paper considers stocks issued by four leading emerging stock markets of the BRIC countries (Brazil, Russia, India, and China) and the United States. The United States are considered in the paper since the country constitutes a major economy and negative shocks are transmitted throughout the financial systems around the world potentially harming the global financial stability. BRIC are of interest because BRIC are among the top 10 countries with the largest gold reserves in the world. China and India together account for around 40% of the total world gold bar and coin demand (World Gold Council, 2020). Russia has emerged as a major gold mining nation, and its central bank over time has built very substantial gold reserves.

To address the issues above we proceed in two stages. In the first step we consider four precious metals, (namely, gold, silver, platinum, and palladium) and analyse if these commodities have offered portfolio diversification opportunities during the first wave of the Covid-19 outbreak in 2020 and how diversification properties (if any) changed across investment horizons. With this target in mind, we propose a novel methodology that combines the benefits of wavelet series expansions with the quantile estimation. We name it "wavelet quantile correlation" procedure, in short, "WQCOR". The procedure can easily be carried out in two steps. The first stage involves using wavelet analysis to decompose the series of precious metals and stock market returns into components associated with different scale resolution. In the second step, the decomposed series are used as input variables to estimate the conditional quantile dependence between the precious metals and the stock markets under consideration. We are particularly interested in estimating how the conditional correlations change over the quantiles of the precious metal distributions in order to investigate the effect of stock market shocks during the first wave of the Covid-19 pandemic outbreak.

Having evaluated the characteristic features of precious metals as potential portfolio diversifiers over different investment horizons, we proceed with the second stage of our investigation and consider how these properties changes over time, once again across investment horizons. With this target in mind we follow Fernandez-Macho

(2012) and use the decomposed series of precious metals and stock market returns to investigate the dynamic patterns of this relationship by calculating rolling window wavelet correlations between the variables of interest. In this case the co-movement dynamics across different investment horizons (time scales) are analysed over time by using weighted window coefficients. The methodology involves estimating a local movement multiple regression to calculate the correlation maps.

This paper contributes to the literature in several ways. First, it conducts an extensive analysis on the relationship between precious metals and stock markets during the first wave of the Covid-19 pandemic outbreak. Considering precious metals as an asset class, this work complements the literature by offering evidence that the correlation between these commodities and stock markets changes across quantiles and investment horizons. Second, unlike the related literature we extend the analysis to a number of precious metals. Most papers consider only the investment and diversification properties of gold. However, the use and the economic drivers of gold markets are different from those of other markets (see Batten *et al.*, 2010; Beckmann *et al.* 2015; Baur and McDermott, 2010; Ciner, 2001). Gold is overwhelmingly used for investment, whereas the other precious metals are heavily used in industry. The question of the ability of silver, platinum, and palladium to provide portfolio diversification opportunities is still open. In the light of these considerations, this paper adds to the related literature by extending the analysis to a relatively large number of precious metals. The proposed methodological approach is the third contribution of the paper. The main innovation of the suggested procedure is the combination of wavelet analysis with the conditional quantile correlation. Wavelet analysis is a filtering method closely related to Fourier analysis and frequency domain methods that transforms the original data into different frequency components with a resolution matched to its scale. Unlike time series and spectral analysis, which only provides information on time-domain and frequency domain respectively, the wavelet method decomposes the financial time series with respect to both time and frequency domains simultaneously. This allows us to investigate if precious metal returns respond differently over short, medium, and long investment horizons. The analysis of conditional quantile correlation between precious metals and stock markets allows us to investigate the impact of stock market shocks on precious metals returns for all portions of the precious metal probability distribution across a wide number of investment horizons. In this respect, the paper also contributes to an emerging literature that seeks to provide a broad perspective on dependence by modelling the relationship between quantiles (see for example Mensi *et al.*, 2016).

The remainder of this paper is organised as follows: Section 2 presents some background on the role of precious metals in portfolio diversification. Section 3 presents the wavelet quantile correlation procedure and the empirical results. Section 4 presents the dynamic correlation analysis. Section 5 presents the implication for portfolio diversification. Section 6 concludes the remarks.

2. Literature Review

The quest for portfolio diversification benefits has attracted a great number of empirical works on safe-haven and hedging properties of gold. The commodity is often analysed in the literature as a candidate for safe-haven during period of financial turmoil. For example, McCown and Zimmerman (2006) estimating a CAPM type model find that gold is zero-beta asset. Baur and Lucey (2010) are the first to formulate an empirically testable

definitions for gold as a “hedge” or “safe haven” assets with respect to other assets such as bonds or stocks. The authors define a hedge as an asset that is uncorrelated with another asset or portfolio on average, whereas a “safe haven” is defined as an asset that is negatively correlated only in times of market turmoil; see also Baur and McDermott (2010). Capie *et al.* (2005) investigates the role of gold as a hedge against the dollar and found a negative relationship between gold and other foreign exchange rates havens (see also Upper, 2000; Kaul and Sapp, 2007). Coudert and Raymond (2010) find that gold is negatively correlated with stock markets during bear periods but not in the long-run. Similarly, Ciner *et al.* (2013) find that gold has a safe haven property against exchange rates in both the United States and the United Kingdom. In addition to gold as a commodity, gold assets such as gold stocks and gold derivatives have also been examined as hedge and safe haven by Jaffe (1989) (see also Worthington and Pahlavani, 2007; Pullen *et al.*, 2011).

If the role of gold in portfolio diversification has been widely considered in empirical works, the literature on the properties of the other three precious metals’ potential is still relatively scarce. For example, Hillier *et al.* (2006) found that silver and platinum prices have good portfolio balancing properties since they estimate that the metals are not correlated with stock market indices (see also Agyei-Ampomah *et al.*, 2013; Mackenzie and Lucey, 2013). Further, Daskalaki and Skiadopoulos (2011) find that the returns of silver, platinum and palladium, have low correlations with stock returns. Morales and Andreosso-O’Callaghan (2011) find that the precious metals markets were less affected by the recent global financial crisis than other major financial markets. Erb and Harvey (2006) and Roache and Rossi (2009) also find that gold and silver prices are counter-cyclical, implying that other precious metals in addition to gold may also protect investors’ wealth during period of market turmoil (see also Sensoy, 2013).

From the modelling perspective most investigators typically adopt GARCH type models to analyse the relationship between precious metals and stock markets. For example, Xu and Fung (2005) used a GARCH type model to examine the shock transmission between the US and Japanese markets for precious metals future contracts. Tully and Lucey (2007) have examined the effect of macroeconomic shocks on gold prices with APGARCH models. Using a DCC-GARCH model Joy (2011) find that gold act a hedge against the US dollar but provide no evidence of gold being the safe haven for US dollar (see also Creti, 2014; Salisu *et al.*, 2020).

The use of wavelet decomposition analysis is relatively recent but is rapidly increasing in the related literature. For example, Bhatia *et al.* (2000) use a hybrid wavelet-based dynamic conditional correlation approach to study the relationship between precious metals and stock markets in the BRICS and G7 countries in the time-frequency domain. He *et al.* (2017) suggest to use multivariate mode decomposition to identify the noise factors in the multiscale domain and forecast the precious metal price movement in order to improve forecasting accuracy of precious metal prices (se also Das *et al.*, 2018; Yoon, 2017). Oral and Unal (2017) use the wavelet coherence method to analyse the forecast co-movement of precious metal.

On the methodological front there are other important studies that examined the relationship between precious metals and other assets, like stock and bonds, using quantile-based approaches. These include Baur and Lucey (2010), Mensi *et al.* (2014), Iqbal (2017) and Adewuyi *et al.* (2019). An interesting model is suggested in Al-Yanyaee *et al.* (2020) where a copula quantile-on-quantile regression is used to examine the correlation between precious metals at different quantiles. The procedure allows the investigator to accommodate for all market conditions (i.e. tranquil, normal and turmoil periods) and exploit portfolio diversification opportunities.

3. The Wavelet Quantile Correlation Procedure

The proposed WQCOR procedure can easily be carried out in two steps. In the first step the Maximal Overlap Discrete Wavelet Transform (MDWT) is applied to the stock market and precious metal returns in order to decompose the series into high-frequency and low-frequency components. In the second step, the obtained filtered series are used as input variables to investigate the portfolio diversification properties of precious metals by calculating the conditional quantile correlation. The two-step procedure to estimate the quantile correlation in the time-frequency domain is described below in more details.

Step 1: The Wavelet Series Expansion

The first step for implementing the WQCOR procedure involves applying the wavelet series expansion to the return series. Wavelet is a technique that decomposes a time series into different short waves that start at a given point in time and end at a given later point in time. In other words, the wavelet approach is a non-parametric method that involves using small wave functions to approximate fluctuations time series to extract information from a sequence of numerical measurements (signals). Broadly speaking, the wavelet decomposition methodology involves applying recursively a succession of low-pass and high-pass filters to the precious metal and stock market series. This process allows separating the high frequency components of the series from the low frequency components (for more details see, for example, Benhmad, 2013). Mathematically, the decomposition of the series in different components can be obtained using wavelet transform which is based on two filters. These are respectively called “mother wavelet” and “father wavelet”. The former is useful to capture the detailed (high frequency) parts of the signal whereas the latter gives information on the smooth (low-frequency) part of the signal. The “father wavelet” (or scaling function) integrates to 1 and is given by

$$\int \phi(t)dt = 1,$$

whereas the mother wavelet integrates to zero and is given by

$$\int \psi(t)dt = 0.$$

Since the use of wavelets is a well-established methodology, in this section we only introduce the concepts and definitions useful for our purposes. For an excellent review of the theory and use of wavelets, see Percival and Walden (2000); Gençay *et al.* (2002).

Let the $f(t) \in L^2(\mathbb{R})$ be a function (for $t = 1, \dots, T$) the time dimensions can be expressed as a linear combination of a wavelet function

$$f(t) = \sum_k s_{j,k} \phi_{j,k}(t) + \sum_k d_{j,k} \psi_{j,k}(t) + \sum_k d_{i-1,k} \psi_{j-1,k}(t) + \dots + \sum_k d_{1,k} \psi_{j-1,k}(t), \quad (1)$$

where the orthogonal basis functions $\phi_{j,k}$ and $\psi_{j,k}$ are defined as

$$\phi_{j,k} = 2^{-j/2} \phi\left(\frac{t-2^j k}{2^j}\right),$$

$$\psi_{j,k} = 2^{-j/2} \psi\left(\frac{t-2^j k}{2^j}\right).$$

In Eq. (1) the representation j is the number of multi-resolution components or scales, and $s_{j,k}$ are the smooth coefficients, and $d_{j,k}$ are called the detailed coefficients. They are approximated by the following integrals

$$s_{j,k} = \int f(t) \phi_{j,k}(t) dt, \quad (2)$$

$$d_{j,k} = \int f(t) \psi_{j,k}(t) dt \quad \text{for } j = 1, 2, \dots, J. \quad (3)$$

The wavelet functions $\phi_{j,k}(t)$ and $\psi_{j,k}(t)$ are scaled and translated version of ϕ and ψ . The smooth coefficient 2^j control the amplitude of the wavelet window so the wavelet function is stretched or compressed to obtain frequency information. Since the scale factor is an exponential function when j gets larger so does 2^j and the functions $\phi_{j,k}(t)$ and $\psi_{j,k}(t)$ become more spread out and shorter. Therefore, a wider window gives information on the low frequency movements, whereas as narrower windows we get information on the high-frequency movements.

As shown by Bruce and Donoho (1996) if the wavelet coefficients can be approximated by the integral in Eq. (2) and Eq. (3) then a multi-resolution representation in Eq. (1) can be simplified

$$F(t) = S_j + D_j + D_{j-1} + \dots + D_j + \dots + D_1, \quad j = 1, \dots, J \quad (4)$$

where D_j is the j -th level wavelet and S_j represents the aggregated sum of variations at each detail of the scale.

In Eq. (1) and Eq. (4) the father wavelet reconstructs the smooth and low-frequency parts of a signal, whereas the mother wavelet function describes the detailed and high-frequency parts of a signal. Therefore, the expression in Eq. (4) provides a complete reconstruction of the time series partitioned into a set of j frequency components so that each component corresponds to a particular range of frequencies.

In the literature several variations of wavelet transform in Eq. (4) have been proposed (see for example Cohen, 1992). In this paper we consider the Maximal Overlap Discrete Wavelet Transform (MODWT). The MODWT has the advantage that the estimated wavelet and scaling coefficients are translation invariant to circularly shifting in the sense that they do not change if the series are shifted in a circular fashion and the smooth coefficients are associated with zero phase filters (for details see Percival and Walden, 2000; Gencay, 2002).

Step 2: The quantile correlation

The second step of the suggested procedure involves using the filtered series obtained from the γ -level multi-resolution decomposition to estimate the conditional quantile correlation.

Let X_m and Y_r be the set of m precious metals (for $m = 1, \dots, 4$) and r stock markets (for $r = 1, \dots, 5$), respectively. We define we define Q_{τ, Y_r} be the τ^{th} unconditional quantile of Y_r and $\tilde{Q}_{\tau, Y_r} = Q_{\tau, Y_r} | X_m$ the τ^{th} conditional quantile of Y_r on X_m . Following Li *et al.* (2015) we define the quantile covariance as

$$qcov_{\tau}\{X_m, Y_r\} = cov\{I(X_m - Q_{\tau, X_m} > 0), Y_r\} = E\{\omega_{\tau}(X - Q_{\tau, X})(Y - E(Y))\},$$

where the function $\omega_\tau(q) = \tau - I(q < 0)$. Accordingly, the quantile correlation between X_m and Y_r is defined as

$$qcor_\tau\{X_m, Y_r\} = \frac{qcov_\tau(Y_r, X_m)}{\sqrt{Var\{\omega_\tau(Y_r, X_m)\}Var(X_m)}} = \frac{E(\omega_\tau(X_m - Q_{\tau, X_m})(Y_r - E(Y_r)))}{\sqrt{(\tau - \tau^2)\sigma_{Y_r}^2}}, \quad (5)$$

where $\sigma_{Y_r}^2 = Var(Y_r)$.

Consider the sample τ^{th} quantile of $Y_{1,r}, \dots, Y_{n,k}$ given by

$$\hat{Q}_{\tau, X_m} = \inf(x_m: F_n(x_m) \geq \tau),$$

where $F_n(X_m) = n^{-1}I(X_{i,m} \leq x_m)$ is the empirical distribution function. Given Eq. (5) the sample estimate of the quantile correlation is given by

$$\widehat{qcor}_\tau(X_m, Y_r) = \frac{1}{\sqrt{(\tau - \tau^2)\hat{\sigma}_{Y_r}^2}} \cdot \frac{1}{n} \sum_{i=1}^n \omega_\tau(X_{i,m} - \hat{Q}_{\tau, X_m})(Y_r - \bar{Y}_r), \quad (6)$$

where $\bar{Y}_r = n^{-1} \sum_{i=1}^n Y_{i,r}$ and $\hat{\sigma}_{Y_r}^2 = n^{-1} \sum_{i=1}^n (Y_{i,r} - \bar{Y}_r)^2$. Li *et al.* (2015) show that, under regularity conditions, the expression in Eq. (6) is a consistent estimator of the quantile correlation and that $\sqrt{n}(\widehat{qcor}_\tau(X_m, Y_r) - qcor_\tau(X_m, Y_r))$ converges in distribution to a $N(0, \Omega)$.

We can use the information provided by the WQCOR procedure to investigate if gold and the other precious metals can be used as asset classes by investors to balance investment portfolios during times of high market volatility. With this target in mind, we follow the definition adopted in Baur and Lucey (2010) (see also Baur and McDermott, 2010) and define an asset as a safe-haven if it is uncorrelated or negatively correlated with another asset or portfolio in times of extreme market movements. Alternatively, an asset is defined as a hedge if it is uncorrelated or negatively correlated with another asset or portfolio on average. The crucial distinction between these two features is that dependence is required to hold under extreme market movements for a safe haven, whereas, for a hedge, it must do so on average. Baur and McDermott (2010) draw a distinction between strong and weak hedges and safe havens on the basis of the negative or null value of the correlation, respectively.

Accordingly, for the m precious metal to be classified as safe haven the estimated correlation coefficient between the lower quantiles of the m precious metal and the r stock market has to be negative. Similarly, to be classified as hedge, the estimated correlation coefficients around the median quantiles of the m precious metal and the r stock market have to be negative or zero. We can thus formulate two hypotheses to determine whether the m metal can serve as a hedge or as safe haven against stock prices:

Hypothesis 1. X_m is a safe-haven if $qcor_\tau(X_m, Y_r) < 0$ for $\tau \leq \tau_0$

Hypothesis 2. X_m is a hedge if $qcor_\tau(X_m, Y_r) \leq 0$ for $\tau_0 < \tau \leq \tau_1$

where $\tau_0 = 0.3$ and $\tau_1 = 0.6$. Note in Hypothesis 2 unlike the definition in Baur and McDermott (2010) where the media is considered, we slightly modify the criterion by taking into consideration the quantiles around the

median. The rationale of doing so is that the second moment of a distribution is notoriously affected by outliers, whereas the median does not have this drawback and seems to be more appropriate for our application.

Note also that Hypothesis 1 is in practice a test on the measure of dependence between the upper tail of X_m and the lower tail of Y_r , which is the probability

$$\text{Hypothesis 1: } X_m \text{ is a safe haven if } \lim_{\tau_0 \rightarrow 1} \Pr(X_m > Q_{\tau_0, X_m} | Y_r < Q_{Y_r(1-\tau_0), Y_r})$$

Likewise,

$$\text{Hypothesis 2: } X_m \text{ is a hedge if } \lim_{\tau_1 \rightarrow 1} \Pr(X_m > Q_{\tau_1, X_m} | Y_r < Q_{Y_r(1-\tau_1)})$$

In addition to Hypotheses 1 and 2, extreme upward market movements of X_m and Y_r may also be of interest, since in this case investors may want extreme upward movements to be positively correlated. Therefore, the following hypothesis may be tested

$$\text{Hypothesis 3. } X_m \text{ is a strong hedge if } qcor_\tau(X_m, Y_r) > 0, \text{ for } \tau > \tau_1$$

Or alternatively,

$$\text{Hypothesis 3. } X_m \text{ is a strong hedge if } \lim_{\tau_0 \rightarrow 1} \Pr(X_m < Q_{\tau_0, X_m} | Y_r > Q_{Y_r(1-\tau_0)})$$

Under hypothesis X_m is a strong-hedge if high positive returns are correlated to high positive returns of Y_r . Note that according to the efficient market hypothesis positive correlation of extreme upward movement between two assets should not last long since it should be impossible to beat the market consistently on a risk-adjusted basis and market should react to new information (see for example Fama, 1970).

3.1 Data and Empirical Results

The data relate to daily stock market closing indices from 20th December 2019 until 15th July 2020 of four major emerging economies, namely Brazil, Russia, India, China. The stock market indexes we consider are the IBV index for Brazil, MOEX for Russia, NIFTY50 for India, SSE for China, and the S&P500 (for the U.S) as a proxy of the largest economy in the world. In addition, the dataset includes prices for four precious metal indices, namely gold, silver, platinum, and palladium. Note that the data under consideration are denominated in US dollars in order to facilitate the comparison between stock market indices and safe haven assets. Following the literature, stock returns are calculated as the difference of the logarithm of the price index.

Table 1 reports, for each time scale the estimated conditional quantile correlations between the X_m precious metal and the Y_r stock market. The decomposition of the r stock and m precious metal returns were obtained applying the MODWT with a the Daubechies compactly supported least asymmetric (LA) wavelet filter of length $L = 6$ (Daubechies, 1992). This filter has been successfully used in several empirical studies to capture the characteristic feature of the financial data (see Gallegati, 2012 and the references therein).

Note that for all families of Daubechies compactly supported wavelets the level j wavelet coefficients are associated with changes at the $\lambda_j = 2^{j-1}$. However, since the MODWT utilizes approximate ideal band-pass filters with bandpass given by the frequency interval $[2^{-(j+1)}, 2^{-j})$ for $j = 1, \dots, J$, inverting the frequency range we have that the corresponding time periods are $(2^j, 2^{j+1}]$ time units (Fernandez-Macho, 2012). In our case since we have 20 daily data for each series per month, we set the scale λ_j with $j = 6$, the highest frequency component D_1 represents short-term variations corresponding wavelet coefficient for intraweek scales 2-4 days, D_2 accounts for variations at a time scale of 4-8 days, near the working days of a week. Similarly, D_3 and D_4 components represent the variations at time scale of 8-16 (approximately fortnightly) and 16- 32 days (approximately monthly scale), respectively. Finally, D_5 and D_6 components represent the long-term variations at time scale of 32-64 (approximately monthly to quarterly scales) and 64-128 days (quarterly to biannual scales). S_6 is the residual of original signal after subtracting D_1, D_2, D_3, D_4, D_5 and D_6 .

In Table 1, the correlation between precious metals and stock returns are reported. The second column report the τ -quantile, whereas the metal and the stock market under consideration are reported in the first column and the first row, respectively. Note that in Table 1, the quantile correlation does not enjoy the symmetric property, this is because the measure of tail dependence through conditional quantiles is dependent on the order of the variables and thus on the non-exchangeability between the variables. This implies that it may be possible that the conditional distribution of the Y_r stock market given X_m precious metal displays tail dependence, whereas $X_m|Y_r$ does not display tail dependence.

Considering now Hypothesis 1, looking at the top part of Table 1A it appears that for most stock markets gold is better able to serve as safe-haven than for hedging since most of the correlation in the lower quantiles are negative, especially in the medium long scales (investment horizons). It is interesting to note that the estimate correlation coefficient is rather stable across different investment horizons since the sign the negative sign persists up to time scale D_6 . Coming to silver, the picture changes since it appears that the ability of silver to act as safe-haven is more scale dependent with medium, low-frequency bands (long horizons) performing better than high-frequency ones (short horizons).

Coming now to platinum and palladium in Table 1B there is evidence that they both enjoy the safe-haven property since correlation in the lowest quantile are mostly negative. It also appears that performance is not scale dependent since the correlation is negative no matter the frequency scale. From Table 1A, it appears that there is not noticeable difference in term of correlation signs between the BRIC stock markets and the United States.

Coming now to Hypothesis 2, in Table 1A it is clear that gold may offer diversification opportunity to investors mainly to medium or long run horizons, since the estimated correlation coefficients for D_4 - D_6 are negative or close to zero. Looking now at silver, we observe negative correlation are more country dependent and scale dependent since the estimated correlation coefficients are negative for IMOEX in scales D_2 - D_5 , however IBV enjoys negative coefficients for scales D_3 and D_4 . On the opposite, NIFTY has negative correlation throughout all the time scales, but D_1 and D_5 . The estimation results for the other precious metals indicate that platinum offers good hedge opportunity for some stock markets such as IBV and SEE where correlations are negative for all time scales (IBV) or all but D_1 and D_4 (SSE). For other stock markets the situation is more mixed with platinum acting as hedge for IMOEX for scales D_3 - D_5 and SP and NIFTY with only two time horizon opportunity.

Finally, the performance of palladium is, once again market dependent, with the precious metal performing very well for some markets such as SSE and IMOEX, and poorly for others such as NIFTY where correlation are negative for the medium time horizons only.

Coming now to Hypothesis 3, this corresponds to days of market exuberance where both metals and stock markets enjoy strong positive returns. Therefore, in this case investors are interested in the positive correlation since negative correlation would imply a loss for a portfolio. Looking at the results in Table 1A, it appears that gold offers good prospects since correlation are mostly positive, no matter the time scale under consideration. The situation for the other metals is more mixed with metals offering good opportunities for some markets, but not for others.

Table 1A: Quantile Correlation

	τ	SSE	IMOEX	IBV	NIFTY	SP	SSE	IMOEX	IBV	NIFTY	SP
<i>Gold</i>	<i>D1 (2-4 days)</i>						<i>D2 (4-8 days)</i>				
	0.1	-0.012	-0.046	-0.026	-0.041	-0.001	-0.074	0.029	-0.044	0.026	-0.027
	0.2	-0.052	-0.062	-0.019	-0.069	-0.030	-0.048	0.038	0.054	0.044	-0.160
	0.3	0.004	0.061	-0.081	-0.071	-0.053	-0.018	0.050	-0.097	0.056	-0.121
	0.4	0.006	0.034	-0.009	-0.099	-0.074	0.005	0.061	-0.079	0.068	-0.098
	0.5	-0.025	0.131	0.097	-0.122	0.033	0.091	0.078	-0.067	0.083	-0.080
	0.6	-0.059	0.077	0.074	-0.124	0.009	0.059	0.096	-0.050	0.010	-0.065
	0.7	-0.071	0.055	0.035	0.085	0.080	0.057	0.119	-0.039	0.049	-0.051
	0.8	0.089	0.019	0.033	0.070	0.062	0.008	0.042	-0.033	0.036	-0.038
	0.9	0.014	0.040	0.038	0.043	0.041	-0.037	0.027	-0.024	0.024	-0.027
	<i>D3 (8-16 days)</i>						<i>D4 (16-32 days)</i>				
	0.1	-0.126	-0.029	-0.028	-0.031	-0.027	-0.015	-0.060	-0.078	-0.130	-0.060
	0.2	-0.110	-0.244	-0.213	-0.065	-0.040	-0.014	-0.067	0.026	-0.090	-0.068
	0.3	-0.176	-0.012	0.027	-0.012	-0.053	-0.014	0.075	-0.016	-0.059	-0.033
	0.4	0.056	0.100	-0.064	0.102	-0.065	-0.006	0.036	-0.067	-0.026	-0.014
	0.5	0.134	0.078	-0.081	0.076	-0.080	-0.014	-0.001	-0.067	-0.026	0.048
	0.6	0.107	0.000	0.000	0.000	0.065	0.004	-0.001	-0.104	-0.063	0.144
	0.7	0.152	0.051	0.052	0.049	0.053	0.007	-0.038	-0.083	-0.050	0.117
0.8	0.103	0.038	0.039	0.036	0.040	0.018	-0.088	0.028	0.014	0.089	
0.9	0.042	0.025	0.026	0.023	0.027	0.013	-0.178	0.060	0.059	0.060	
<i>D5 (32-64 days)</i>						<i>D6 (64-128 days)</i>					
0.1	-0.063	-0.071	-0.051	0.060	-0.071	-0.019	-0.055	-0.155	-0.048	-0.075	
0.2	-0.101	-0.107	-0.091	-0.008	-0.107	-0.091	-0.082	-0.050	-0.092	-0.102	
0.3	-0.048	-0.074	0.003	0.012	-0.054	0.015	-0.108	0.008	0.011	0.069	
0.4	-0.017	-0.080	0.067	0.066	-0.029	-0.026	-0.132	-0.024	0.081	0.071	
0.5	-0.015	-0.048	0.023	0.066	-0.048	0.022	-0.162	-0.027	0.046	0.036	
0.6	-0.046	-0.129	-0.020	0.031	-0.092	0.021	-0.199	-0.006	0.014	0.004	
0.7	0.190	-0.066	-0.065	-0.011	-0.052	0.089	-0.008	0.070	-0.018	0.028	
0.8	-0.131	0.066	0.119	0.066	0.007	0.058	0.050	0.082	0.072	0.062	
0.9	-0.087	-0.026	-0.179	-0.026	-0.062	0.053	-0.122	0.055	0.045	0.035	
<i>Silver</i>	<i>D1 (2-4 days)</i>						<i>D2 (4-8 days)</i>				
	0.1	-0.038	-0.046	-0.038	-0.142	-0.039	-0.069	-0.056	-0.024	-0.033	-0.023
	0.2	0.092	-0.062	-0.051	-0.078	-0.057	-0.132	-0.136	-0.029	-0.078	-0.031
	0.3	0.059	-0.078	-0.075	-0.043	-0.085	-0.103	-0.010	-0.020	-0.079	-0.047
	0.4	0.069	-0.092	-0.078	0.012	0.082	0.082	-0.082	-0.029	-0.102	-0.103
	0.5	0.131	-0.113	-0.128	0.005	0.033	0.069	-0.061	-0.044	-0.110	-0.068
	0.6	0.184	-0.138	-0.184	0.092	0.140	0.059	-0.053	-0.055	-0.033	-0.052
	0.7	-0.027	-0.171	-0.054	0.048	0.078	-0.052	-0.038	-0.100	-0.013	-0.038
	0.8	-0.019	0.096	0.057	0.070	0.093	-0.014	-0.032	0.135	0.000	-0.029
	0.9	-0.023	0.027	0.081	0.031	0.038	-0.034	-0.013	0.235	0.004	-0.015
	<i>D3 (8-16 days)</i>						<i>D4 (16-32 days)</i>				
	0.1	-0.051	-0.018	-0.018	-0.034	0.047	-0.073	-0.078	-0.023	-0.133	-0.033
	0.2	-0.081	-0.051	-0.051	-0.051	0.010	-0.001	-0.037	-0.042	-0.049	-0.017
	0.3	-0.071	-0.013	-0.013	-0.066	-0.013	-0.026	-0.056	-0.034	-0.064	-0.054
	0.4	0.102	-0.123	-0.123	-0.082	0.123	-0.043	-0.69	-0.041	-0.079	-0.090
	0.5	0.046	0.100	-0.100	-0.100	0.100	-0.089	-0.088	-0.059	-0.069	0.069
	0.6	-0.004	0.082	0.082	-0.123	0.082	-0.029	0.110	-0.081	-0.090	0.050
	0.7	-0.005	0.066	0.066	-0.152	0.066	-0.027	-0.137	-0.106	0.064	0.034
0.8	0.007	0.051	0.051	-0.139	0.051	0.112	-0.182	-0.147	0.049	0.014	
0.9	0.001	0.034	0.034	-0.219	0.034	0.135	-0.228	0.023	0.133	0.033	
<i>D5 (32-64 days)</i>						<i>D6 (64-128 days)</i>					
0.1	-0.061	-0.071	-0.326	-0.105	-0.064	-0.007	-0.014	-0.042	-0.037	-0.040	
0.2	-0.062	-0.099	-0.197	0.197	-0.096	-0.006	-0.096	-0.111	-0.106	-0.060	
0.3	-0.097	0.050	0.135	0.135	-0.212	-0.010	-0.108	-0.063	-0.058	-0.078	
0.4	-0.017	0.096	0.110	0.138	0.167	-0.013	-0.096	0.003	0.032	-0.096	
0.5	-0.024	0.172	0.069	0.106	0.172	0.108	-0.123	0.002	0.007	-0.118	
0.6	0.072	0.136	-0.136	0.069	0.146	0.061	-0.114	-0.022	-0.017	-0.017	
0.7	0.023	-0.115	0.106	0.035	-0.117	0.037	-0.042	-0.47	-0.042	0.065	
0.8	0.106	0.096	0.073	0.096	0.086	-0.012	-0.072	-0.083	-0.078	0.060	
0.9	0.083	0.064	0.034	0.034	0.041	-0.034	-0.144	0.035	0.040	0.097	

Note: The table reports the estimated quantile correlation coefficients between precious metals and stock markets indexes by scale. To obtain the wavelet coefficients at each scale, the Daubechies extremal phase wavelet filter of length 8 is applied. The method of estimating the quantile correlation is defined in Eq. (6).

Table 1B (Continue): Quantile Correlation

	τ	SSE	IMOEX	IBV	NIFTY	SP	SSE	IMOEX	IBV	NIFTY	SP	
<i>Palladium</i>	<i>D1 (2-4 days)</i>						<i>D2 (4-8 days)</i>					
	0.1	-0.063	-0.038	-0.215	-0.009	-0.019	-0.023	0.162	-0.018	-0.034	-0.016	
	0.2	-0.053	-0.057	-0.154	-0.014	-0.024	-0.011	-0.107	-0.060	-0.211	-0.047	
	0.3	0.064	-0.075	-0.129	0.018	-0.028	-0.081	-0.103	-0.173	0.151	-0.031	
	0.4	0.086	-0.092	-0.115	0.022	-0.032	-0.089	-0.074	-0.187	0.211	-0.043	
	0.5	0.109	-0.113	-0.107	0.028	-0.122	-0.063	-0.092	-0.184	0.175	-0.055	
	0.6	0.015	-0.138	-0.018	0.028	-0.130	-0.097	0.056	-0.220	0.169	0.060	
	0.7	0.023	-0.171	-0.013	0.034	-0.144	-0.046	0.072	-0.099	0.206	0.074	
	0.8	0.042	0.057	-0.014	0.042	-0.164	-0.063	0.038	-0.110	0.213	0.057	
	0.9	0.012	0.038	-0.009	0.149	-0.225	-0.036	0.074	0.006	0.161	0.061	
	<i>D3 (8-16 days)</i>						<i>D4 (16-32 days)</i>					
	0.1	-0.163	-0.014	-0.026	-0.004	-0.052	-0.189	-0.067	-0.102	-0.017	-0.057	
	0.2	-0.181	-0.124	-0.035	-0.005	-0.036	-0.079	-0.014	-0.033	-0.025	-0.044	
	0.3	-0.102	-0.106	0.029	-0.007	-0.029	-0.031	-0.049	-0.004	-0.033	-0.049	
	0.4	-0.014	0.097	0.131	-0.009	0.131	-0.022	-0.056	-0.004	-0.084	0.051	
	0.5	-0.086	0.011	0.127	-0.126	0.128	-0.139	-0.065	-0.014	-0.066	0.023	
	0.6	0.092	0.009	0.126	-0.131	0.126	-0.072	-0.137	-0.025	-0.095	0.013	
	0.7	0.081	0.007	0.133	-0.142	0.113	-0.093	-0.157	-0.055	0.059	0.004	
	0.8	0.021	0.005	0.005	-0.124	-0.153	0.302	-0.192	-0.105	0.055	-0.008	
	0.9	0.074	0.004	0.004	-0.169	-0.136	0.036	-0.229	0.089	0.017	0.013	
	<i>D5 (32-64 days)</i>						<i>D6 (64-128 days)</i>					
	0.1	-0.050	-0.071	-0.079	-0.014	-0.133	-0.046	-0.005	-0.138	-0.005	-0.005	
	0.2	-0.042	-0.011	-0.056	-0.166	-0.006	-0.098	-0.007	-0.171	-0.007	-0.007	
	0.3	-0.053	-0.111	0.037	0.124	-0.092	-0.032	-0.010	-0.069	0.046	-0.010	
	0.4	-0.083	-0.065	0.044	0.051	0.067	-0.077	-0.012	-0.068	0.040	-0.012	
	0.5	-0.067	0.017	0.040	0.054	0.022	-0.154	-0.014	-0.069	0.037	-0.050	
	0.6	-0.036	-0.001	0.092	-0.052	0.006	0.138	-0.018	-0.073	0.035	0.035	
	0.7	-0.027	0.002	-0.003	-0.053	0.003	-0.022	0.034	-0.082	0.034	0.109	
	0.8	0.115	-0.018	0.086	0.007	0.009	0.011	0.035	0.007	0.035	0.033	
	0.9	0.076	0.004	-0.014	0.104	0.133	0.009	0.067	0.038	0.049	0.005	
	<i>Platinum</i>	<i>D1 (2-4 days)</i>						<i>D2 (4-8 days)</i>				
		0.1	-0.018	-0.031	-0.215	-0.019	-0.019	-0.023	-0.100	-0.018	-0.035	-0.147
		0.2	-0.020	-0.014	-0.154	-0.311	-0.099	-0.011	-0.047	-0.083	-0.067	-0.204
		0.3	-0.018	-0.013	-0.129	-0.134	-0.091	-0.100	-0.002	-0.170	-0.004	-0.199
		0.4	0.022	0.024	-0.115	-0.120	-0.081	-0.089	-0.059	-0.192	0.058	-0.143
		0.5	0.112	0.028	-0.107	-0.122	0.132	-0.083	0.048	-0.202	0.078	-0.230
0.6		0.120	0.034	-0.018	-0.022	0.064	-0.081	0.023	-0.243	0.095	-0.207	
0.7		0.134	0.042	-0.013	-0.018	-0.053	-0.036	0.061	-0.182	0.116	-0.243	
0.8		0.154	0.049	-0.014	-0.020	0.023	-0.046	-0.025	-0.167	0.126	-0.264	
0.9		0.215	0.149	-0.029	-0.009	-0.063	-0.011	-0.033	-0.068	0.134	0.132	
<i>D3 (8-16 days)</i>						<i>D4 (16-32 days)</i>						
0.1		-0.191	0.020	-0.06	-0.020	-0.020	-0.136	-0.085	-0.139	-0.027	-0.069	
0.2		-0.144	-0.024	0.029	-0.029	-0.024	-0.182	-0.140	-0.040	-0.040	-0.128	
0.3		-0.142	-0.038	-0.116	-0.106	-0.038	-0.122	-0.085	0.053	-0.054	-0.153	
0.4		-0.134	-0.034	-0.097	-0.110	0.131	0.023	-0.104	0.117	-0.041	-0.065	
0.5		-0.145	-0.022	-0.120	-0.084	0.034	0.123	-0.073	0.180	-0.056	-0.080	
0.6		0.019	-0.010	-0.110	-0.047	0.049	0.149	-0.042	0.098	-0.165	-0.065	
0.7		0.002	0.001	-0.106	-0.038	0.133	-0.079	0.103	-0.053	-0.227	-0.053	
0.8		0.001	0.016	-0.029	-0.029	0.148	-0.022	0.067	-0.040	-0.040	0.089	
0.9		0.003	0.020	-0.016	-0.020	0.217	-0.016	0.103	-0.088	0.029	0.027	
<i>D5 (32-64 days)</i>						<i>D6 (64-128 days)</i>						
0.1		-0.066	-0.031	-0.202	-0.023	-0.129	-0.093	-0.010	-0.017	-0.034	-0.015	
0.2		-0.028	-0.003	-0.079	-0.029	-0.013	-0.091	-0.015	-0.060	-0.015	-0.019	
0.3		-0.073	-0.014	-0.165	0.023	-0.005	-0.106	-0.020	-0.045	-0.020	-0.023	
0.4		-0.150	-0.061	-0.153	0.022	-0.022	-0.109	0.025	-0.037	0.028	0.028	
0.5		-0.061	-0.059	-0.051	0.023	-0.051	-0.054	0.030	-0.030	0.029	0.033	
0.6		0.083	-0.059	-0.050	0.025	-0.006	0.157	0.038	-0.024	0.036	0.040	
0.7		0.084	-0.061	-0.052	-0.005	-0.005	0.116	0.005	-0.019	0.045	0.049	
0.8		0.095	-0.068	-0.058	-0.003	-0.003	0.061	0.065	-0.015	0.059	-0.016	
0.9		0.050	-0.016	-0.075	-0.004	-0.002	0.088	0.010	-0.015	0.070	-0.012	

Note: The table reports the estimated quantile correlation coefficients between precious metals and stock markets indexes by scale. To obtain the wavelet coefficients at each scale, the Daubechies extremal phase wavelet filter of length 8 is applied. The method of estimating the quantile correlation is defined in Eq. (6).

4. Dynamic Wavelet Correlation

The results in Table 1 are informative about the joint behaviour of precious metals and stock markets in different time scales. However, it is also of interest looking at the dynamics of the correlation since it is well known that correlations change over time. To investigate this issue we follow Fernandez-Macho (2018) and use the time-localized multiple regression model to estimate time varying correlation between precious metals and stock markets. The method allows to calculate the set of multiscale correlations along time and across different scales by estimating a series of windowed wavelet coefficients. In particular, let $Z = (X_m, Y_r)$ be a realization of a multivariate stochastic process, and $Z_{-i} = Z/\{z_i\}$ for some $z_i \in Z$. For a fixed $s \in (1, \dots, T)$, Fernandez-Macho (2018) suggest to minimize a weighted sum of the squared errors

$$S_s = \min \sum_t \vartheta(t-s) [f(Z_{-i,t}) - z_{i,t}]^2,$$

where $f(Z_{-i,t})$ is a linear function and ϑ is a given moving average weighted function. The local weighted least squared approximation around s can be written as

$$f(Z_{-i}) = M_i \beta_s, \quad (7)$$

where $M_i = Z_{-i} - \bar{Z}_{-i}$. The β_s can therefore be estimated by calculating

$$\hat{\beta}_s = [\sum_t \vartheta(t-s) M_{i,t} M'_{i,t}]^{-1} \sum_t (t-s) M_{i,t} z_{i,t}. \quad (8)$$

Letting s moving overtime, for the local regression in Eq. (7) local coefficient of determination can be calculated from the weighted sum of the local squared residuals.

In analogy with the classical regression model, at each wavelet scale the wavelet local multiple correlation coefficient can be estimated as

$$\hat{\sigma}_{M,s}(\lambda)_j = \text{Corr}(\vartheta(t-s)^{1/2} \omega_{i,j}, \vartheta(t-s)^{1/2} \hat{\omega}_{i,j})$$

where $\omega_{i,j}$ are the wavelet coefficients chosen to maximise $\hat{\sigma}_{M,s}(\lambda)_j$ and $\hat{\omega}_{i,j}$ are the fitted values in Eq. (8) (for more details, see Fernandez-Macho, 2018) the joint behaviour of precious metals and stock markets in times of extreme markets movements.

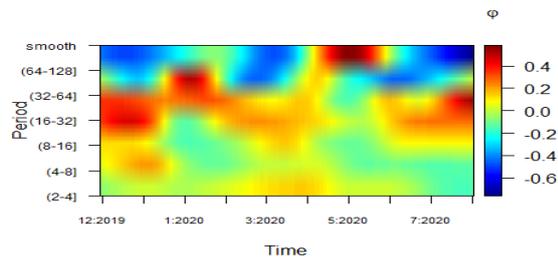
In Figure 1, the correlation patterns between the returns of the precious metals under consideration and the stock market indexes are presented in a time-frequency domain on a scale by scale basis. Therefore, in Figure 1 the correlation coefficients are calculated daily for each pair of stock markets returns and precious metals. For ease of interpretation, the left-hand horizontal axis is transformed to show the number of days in which the scale moves from low to high wavelengths. The heat maps indicate the increasing strength of the correlation among the stock markets indexes as they move from blue (lowest correlation) to red (highest correlation).

Looking at the results in Figure 1 it appears that the time horizon under consideration is quite an important feature when it comes to evaluating the performance of precious metals as portfolio stabilizer in time of market

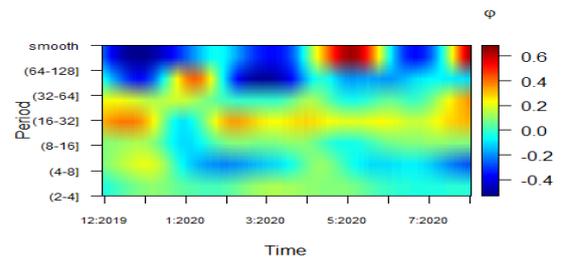
distress. The correlation patterns are also different across stock markets with some market performing better than others.

For example, gold appears to have good diversification property for long horizon investors for SP, Nifty and SSE stock markets, but less so in the short run, since in the short time scale the correlation is positive. Note, however, that a contagion effect emerges for the scale D6 during the high of the Covid-19 outbreak between January 2020 to March 2020 as highlighted by the red colour in Figure 1. On the other side, silver performed well as in the short-medium time scales, thus suggesting that it may serve as portfolio stabiliser for short-medium horizon investors. It is interesting to note that also in this case the period from January 2020 to March 2020 feature positive correlation for IBV and SP for D6 and S6, thus signalling contagion effect between the two markets. Looking now at the palladium and platinum, they perform well in short time scales thus overtaking gold and silver in term of investment opportunity for short-medium horizon investors.

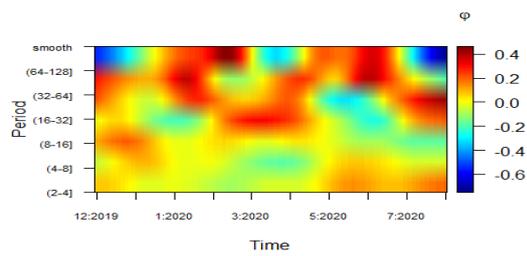
Figure 1: Dynamic wavelet correlation between precious metals and stock markets.



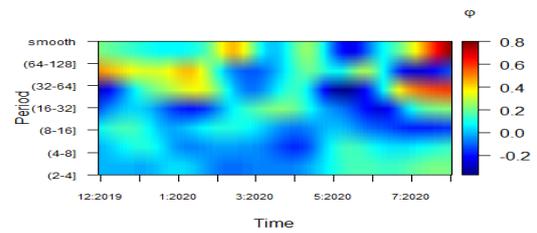
Gold and SSE



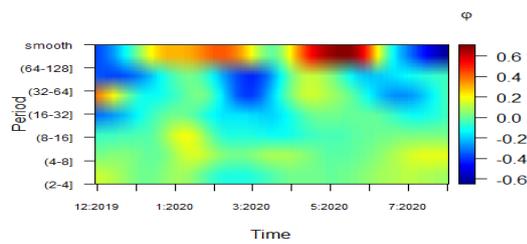
Silver and SSE



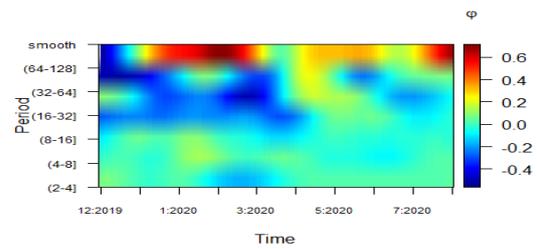
Gold and IMOEX



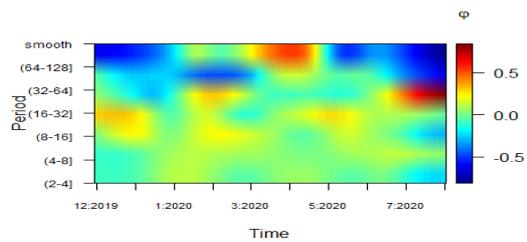
Silver and IMOEX



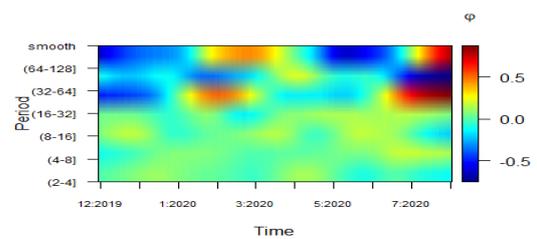
Gold and IBV



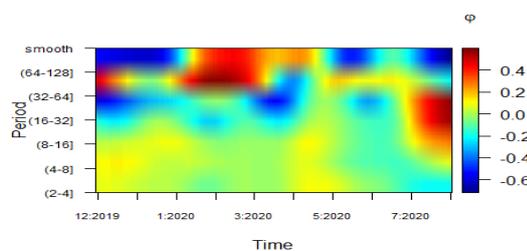
Silver and IBV



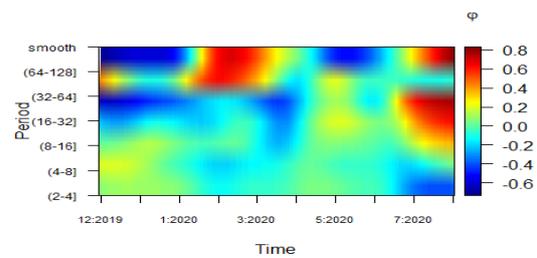
Gold and NIFTY



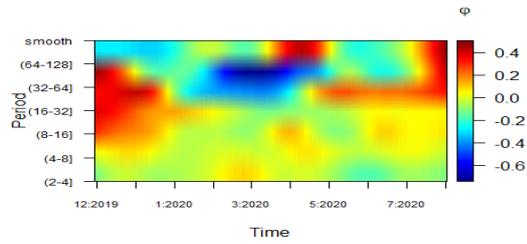
Silver and NIFTY



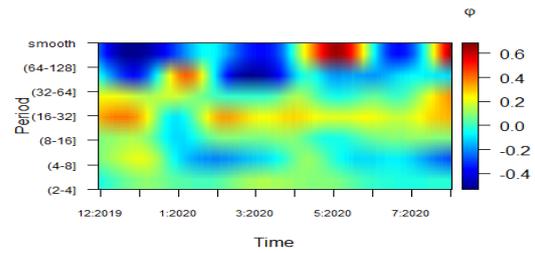
Gold and SP



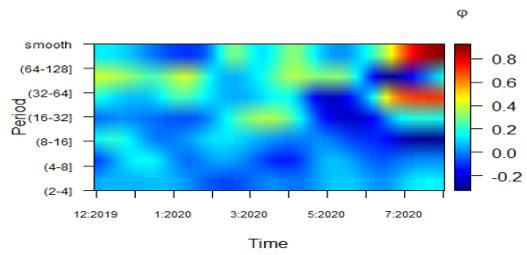
Silver and SP



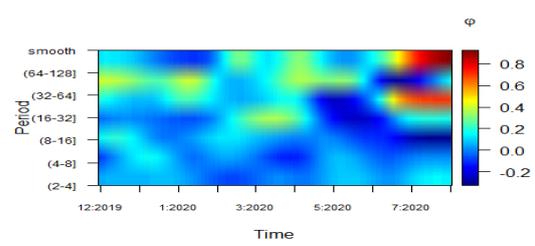
Palladium and SSE



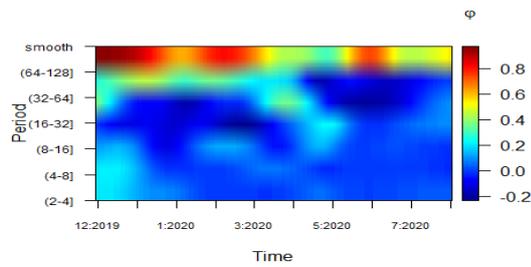
Platinum and SSE



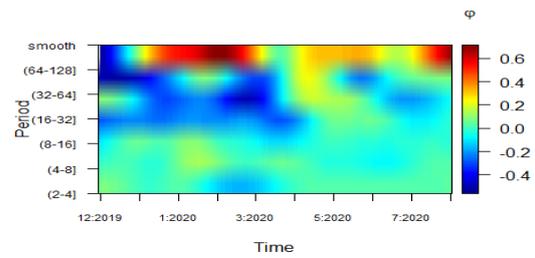
Palladium and IMOEX



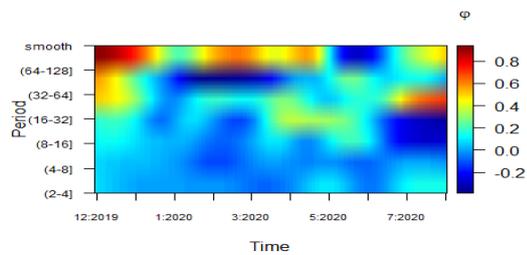
Platinum and IMOEX



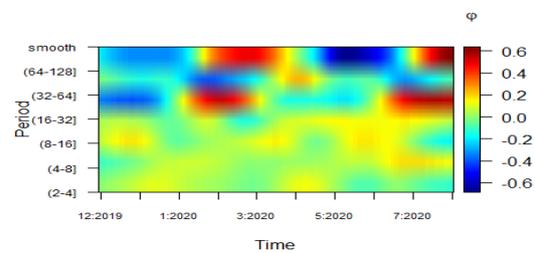
Palladium and IBV



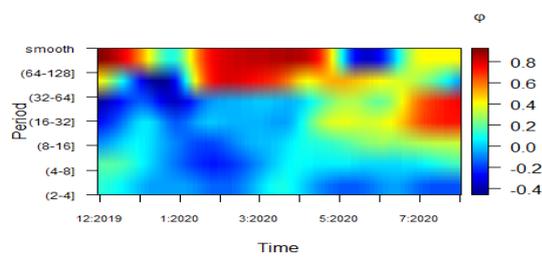
Platinum and IBV



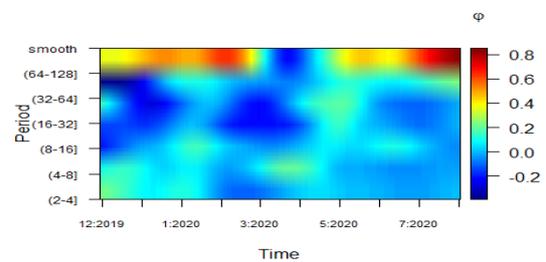
Palladium and NIFTY



Platinum and Nifty



Palladium and SP



Platinum and SP

Overall, the variety of wavelet correlation patterns displayed in Figure 1 two main findings emerge: *i*) there is evidence precious metals offered portfolio diversification opportunities during the Covid-19 health crisis; *ii*) the diversification properties are property is scale dependent, since negative correlation has been observed in the short-medium frequency for platinum and palladium for most stock markets, whereas the opposite is true for gold and silver where spells of negative correlation are observed in the long-run only.

5. Implications for Portfolio Construction

The analysis in the previous sections provide several insights on the dependence structure between stock markets and precious metals. In particular, the analysis in Section 3 allows the investigator to answer the question: How does the correlation of the τ -quantile of the m -metal conditional on the r -stock return changes when the returns of r -stock market changes? In this section we are interested in answering the question: What are the implications for risk management and portfolio construction strategies?

To examine this issue we compute optimal portfolio weights and hedge effectiveness of m precious metals for the r stock index under consideration using the conditional variance and covariance estimates obtained using the WQCOR procedure. With this target in mind we estimate the optimal portfolio weights to evaluate the optimal proportion of the precious metals and the equities that should form a rational investor's portfolio. To evaluate the optimal portfolio weight, for each quantile and across investment horizons we propose a variation of the approach proposed by Kroner and Ng (1998). An investor in a metal m , willing to hedge against adverse price movements in the stock portfolio r without short selling, can decide their portfolio according to the following formula:

$$\bar{\omega}_{mr} = \frac{Var(X_m) - Cov(X_m Y_r)}{Var(X_m) - 2Cov(X_m, Y_r) - Var(Y_m)}, \quad (9)$$

$$\bar{\omega}_{mr} = \begin{cases} 0, & \text{if } \bar{\omega}_{mr} < 0 \\ \bar{\omega}_{mr} & \text{if } 0 < \bar{\omega}_{mr} \leq 1 \\ 0, & \text{if } \bar{\omega}_{mr} > 1 \end{cases}$$

where $\bar{\omega}_{mr}$ is the weight of the m metal in a one-dollar portfolio of X_m and Y_r at time t , index $Var(X_m)$ and $Var(Y_m)$ are the conditional variance of X_m and of the variance of Y_m , respectively, obtained the WQCOR procedure and $Cov(X_{m,t} Y_{r,t})$ is the conditional covariance between metal m and equity index r . Note that in Eq. (9) the subscripts for the τ -quantile and the j time scale (investment horizon) have been omitted to improve clarity. The weight of X_m is calculated as $(1 - \bar{\omega}_{mr})$.

Table 2 reports the summary statistics of the portfolio weights computed applying the Kroner and Ng (1998) approach to choose the optimal portfolio weights. To safe space the results for gold, silver, SP, and IBV only are presented. Namely, silver is considered as representative of the white metals and the IBV index as benchmark of the BRIC stock markets. Also, the results relate the optimal portfolio weights for scale D2 and D5, that is, short investment and long investment horizons. In the second column the relevant quantiles are described, so that $\tau \leq 0.3$ corresponds to the optimal portfolio weights in case of extreme negative returns in stock markets under consideration. As before, the middle quantiles correspond to hedging during "normal" periods and the top quantiles are related to the optimal portfolio weights in case of extreme positive returns in stock markets.

Table 2. Optimal portfolio weights.

<i>Quantile</i>		<i>Gold</i>				<i>Silver</i>			
		Mean	St. Dev	Min	Max	Mean	St. Dev	Min	Max
<i>D2</i>									
<i>SP</i>	$\tau \leq 0.3$	0.571	0.114	0.238	0.721	0.471	0.123	0.231	0.523
	$0.3 < \tau \leq 0.6$	0.507	0.363	0.258	0.981	0.351	0.114	0.141	0.510
	$0.6 < \tau \leq 0.9$	0.421	0.214	0.219	0.623	0.252	0.101	0.114	0.451
<i>IBV</i>	$\tau \leq 0.3$	0.621	0.157	0.261	0.710	0.526	0.210	0.341	0.712
	$0.3 < \tau \leq 0.6$	0.437	0.189	0.313	0.691	0.412	0.218	0.218	0.691
	$0.6 < \tau \leq 0.9$	0.335	0.134	0.214	0.523	0.345	0.199	0.198	0.519
<i>D5</i>									
<i>SP</i>	$\tau \leq 0.3$	0.489	0.134	0.223	0.651	0.582	0.321	0.323	0.791
	$0.3 < \tau \leq 0.6$	0.413	0.189	0.156	0.637	-	-	-	-
	$0.6 < \tau \leq 0.9$	0.291	0.111	0.159	0.489	-	-	-	-
<i>IBV</i>	$\tau \leq 0.3$	0.751	0.341	0.445	0.851	0.781	0.312	0.423	0.890
	$0.3 < \tau \leq 0.6$	-	-	-	-	-	-	-	-
	$0.6 < \tau \leq 0.9$	-	-	-	-	-	-	-	-

Looking at the results from Table 2 it appears that the optimal portfolio for the gold/SP portfolio for the investment horizon 4-8 days for the $\tau \leq 0.3$ quantile, is 0.577, which indicates that for a \$1 portfolio, nearly 60 cents should be invested in gold and 40 cents in SP500 stock market index. In more normal periods this proportion should be reduced roughly to 50 cents in each asset for the same investment horizon. For longer investment horizons, this percentage reduces considerably no matter the quantile under consideration. Considering now the optimal weights for the silver/SP portfolio the average weights are lower in general for the D2 time scale. This result is in line with the conditional correlation coefficients reported in Table 1 that are in average higher for gold/SP than silver/SP. Coming now to the D5 scale only the optimal weights for the lower quantiles are reported, since according the results in Table 1 the correlation for this time scale are positive, therefore silver does not constitutes a diversifier for Silver/SP stock portfolio. Looking now at the optimal weights for the Gold/IBV portfolio the weight is higher for the lower quantiles but lower for the other quantile. Silver does not look a good diversifier in the long horizon in Table 1, therefore the optimal portfolio weights have been omitted.

Discussion

Before concluding this section a question is in order: What do we learn about the relationship between stock market and precious metals? The results in Table 1 illustrates that the multiscale relationship can be a useful tool for portfolio diversification. Market participants constitute a very heterogenous group and make investment

decisions over different horizons. For example, intraday traders have diverse objective from hedging strategists, international portfolio managers, or large multinational corporations. Since investors have different time horizons, it is natural for them to seek to the minimise portfolio idiosyncratic risk at a given time scale. For example, even in condition of extreme market distress such as Covid19 outbreak, positive correlations in short-run between precious metals and stock markets may not be important to their investment goals to investors with long-run time horizons, such a pension funds for example. However, most previous empirical studies focus on two-scale analysis, namely the short-run and long-run periods. The proposed WCOR procedure allows the investigator to pre-process financial time series of interest using the multiscale decomposition to analyse the conditional quantile correlations at different investment horizons. In this respect, the wavelet may act as a “lens” enabling the investigator to unveil characteristic features of time series that would not be observable using traditional econometric methods.

We are not the first authors to use wavelets to analyse the relation between financial time series variables (see for example Al-Yahyaee, 2019). However, most of the related literature makes use of discrete and continuous wavelet transforms to calculate the tail dependence or the wavelet correlation coefficient between financial series. The WQCOR procedure allows to measure the quantile dependence between time series for each quantile. Therefore, it allows to capture how the relationship changes in each state of the market (e.g. bearish, bullish, or normal). The suggested procedure in the spirit of Mensi *et al.* (2016) (see also Xu *et al.* 2020). where quantile regression models are estimated from the decomposed series obtained using a wavelet analysis It can be shown that

$$qcor_{\tau}\{X_m, Y_r\} = sign\left(\beta_{X_m Y_r}(\tau)\right) \sqrt{\beta_{X_m Y_r}(\tau) \beta_{Y_r X_m}(\tau)}$$

where $\beta_{X_m Y_r}$ are the regression coefficients of the quantile regression of X_m on Y_r , and $\beta_{Y_r X_m}$ are the estimated coefficient of the reciprocal quantile regression (see Choi and Shin, 2018). Therefore, taking the geometric mean of the two τ -quantile regressions delivers similar results to those obtained from the WCOR procedure. However, the proposed procedure is computationally less cumbersome to estimate. Moreover, the upper and the lower tail parameters are notoriously difficult to estimate by maximum likelihood. This is due to the fact that there are relatively fewer observations in the tails of the conditional distributions of returns. The problem can be easily avoided by directly estimating the correlation coefficient between quantiles.

From the results in Figure 1 and Table 1 it is clear that precious metals may play an important role in balancing portfolios. However, not only correlations change over quantiles and investment horizons, but also over time. Therefore, it is crucial for investors to consider the anticipated holding period of precious metals assets in their portfolios. From the econometric point of view, a growing number of empirical applications in the field have found evidence of consistent nonlinear dependencies. For example, Choudhry *et al.* (2015) investigate the nonlinear dynamic co-movements between gold returns and stock market returns and found evidence of nonlinear feedback effect among the variables during the during the global financial crisis period (Kyrtsov *et al.*, 2006). Evidence of nonlinearity corroborate the use of the MODWT since wavelet transform are robust to regime shifts. Actually, one may argue that time-localized wavelet multiple regression approach is particularly fits for purpose, since the methodology is robust to changes in dependence structure of the stochastic processes under investigation,

such as non-stationarity spells of processes for example or any type of changes in the dependence structure of the stock market series we may observe during periods of extreme market distress such as the Covid19 pandemic outbreak. Failing to account for the characteristic feature of the series would resolve in less than optimal investment strategy to investors. Looking forward it would be interesting to use factor models on the decomposed series of precious metals returns and apply exogenous variables able to explain the systematic risk of a portfolio on a scale-by scale basis. This would allow to investigate the effect of systematic risk factors on portfolio diversification on different investment horizons.

6. Conclusion

In this study we investigate the role of precious metals such as gold and silver in portfolio diversification assets during the early period of Covid-19 pandemic. With this target in mind, we propose a novel approach to investigate the dependence structure between precious metals and stock market for different investment horizons by using wavelet decomposition prior to calculate the quantile correlation coefficient. The suggested WCOR procedure identifies the quantiles of metal and stock returns where negative correlation allows to achieve maximum benefit from portfolio diversification for each given investment horizon. At the same time, for each investment horizon, the procedure also allows to spot the quantiles of stock and metal returns at which correlations are positive, thus providing an opportunity to portfolio managers to make informed decisions about when they should avoid going long or short on both the assets under consideration. This gives investors flexibility about the choice of the time and at what investment horizon they should enter the market.

The estimated results reveal that precious metals can be successfully used to balance portfolios even in periods of extreme market distress. In particular, the WCOR procedure revealed that gold can act as safe-haven especially for medium-long run investment horizon (investment strategies 8-16 days and longer), although some evidence of positive quantile correlation between the lower quantiles of the yellow metal and the quantile of IMOEX, NIFTY, and IBV market returns is found for horizons between 4-8 days. Silver is also found to performs better as medium-long run safe haven since pocket of short-run positive correlation between the lower quantile are found for shorter than 8 days. Looking at the median correlation it appears that gold is better able to act as hedge for most stock markets for the short-medium investment horizons, whereas silver better serves as hedge in medium run horizons. Perhaps the most important finding of the paper is that gold and silver, which are the most well-known precious metals, are not the only available commodities to investors since the quantile correlation analysis has revealed that platinum and palladium in several instances overtake the former metals in term of safe-haven and hedging properties. In this respect, our results are in line with Bredin *et al.* (2015) where it was found that gold and gold stocks have been shown to have low levels of correlation with equity indices, highlighting their role as a diversifier (see, for example, Chua, 1990; Hillier *et al.*, 2006).

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