Department of Economics and Statistics "Cognetti de Martiis"
Campus Luigi Einaudi, Lungo Dora Siena 100/A, 10153 Torino (Italy)
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COVID-19 PANDEMIC AND STOCK MARKET CONTAGION: A WAVELET-COPULA GARCH APPROACH

HUTHAIFA ALQARALLEH, ALESSANDRA CANEPA and EMILIO ZANETTI CHINI

UNIVERSITÀ DEGLI STUDI DI TORINO

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COVID-19 Pandemic and Stock Market Contagion:

A Wavelet-Copula GARCH Approach

Huthaifa Alqaralleh*
Mutah University

Alessandra Canepa†
University of Turin
and Brunel University London

Emilio Zanetti Chini‡
University of La Sapienza

Abstract

In this study, we examine the influence of the COVID-19 pandemic on stock market contagion. Empirical analysis is conducted on six major stock markets using a wavelet-copula GARCH approach to account for both the time and the frequency aspects of stock market correlation. We find strong evidence of contagion in the stock markets under consideration during the COVID-19 pandemic.

Keywords: Financial Market Contagion; COVID-19 Pandemic; Wavelet Analysis; Copula GARCH.

JEL classification: C35, F37, G10, G15

* Department of Economics, Business and Finance, Mutah University, Karak, Jordan. E-mail address: huthaifa89@mutah.edu.jo.

† Corresponding author: Department of Economic and Statistics Cognetti De Martiis, University of Turin, Lungo Dora Siena 100A, 10153 Turin, Italy, and Department of Economics and Finance, Brunel University London, Uxbridge, UB8 3PH, United Kingdom. E-mail address: Alessandra.Canepa@unito.it.

‡ Sapienza University of Rome, Department of Economics and Law, Rome, Italy. E-mail address: emilio.zanettichini@uniroma1.it

1. Introduction

The present study looks for fresh insights into the extent to which stock markets have been affected by the COVID-19 crisis, asking whether the apparent market transmission is actually the effect of contagion or interdependence.

To understand the transmission of shocks across stock markets, a two-step investigation has been conducted. In the first step, we analyse the wavelet spectrum for the return series using wavelet multiresolution decomposition. The advantage of such decomposition is that it enables the data analysis to take place at equally spaced intervals (see, for example, Crowley, 2007). In the second step, the filtered series has been used to measure the time-varying correlation dynamics. Motivated by the fact that different financial decisions occur at different frequencies, we examine stock market contagion at different frequencies and identify the timescales in which the benefits of portfolio diversification in terms of risk management are low. To this end, a novel wavelet-copula GARCH analysis is proposed to take into account both the time and the frequency aspects of stock market connectedness. Put differently, disentangling the behaviour of risk at the frequency level can capture its time-varying features. Consequently, both the evolving exposure to risk and the risks faced by short- and long-term investors can be distinguished and measured simultaneously.

The remainder of this study is organized as follows. Section 2 outlines the methodology, whereas Section 3 presents the data and the empirical results. Finally, Section 4 contains some concluding remarks.

2. Methodology

2.1 The Maximum Overlap Discrete Wavelet Transform

Wavelet is a well-established technique that decomposes a time series into small waves that begin at a specific point in time and end at a later specific point in time. A significant advantage of this approach is that frequency information can be obtained without losing the timescale dimension. Another advantage of wavelet analysis is that it does not need to assume anything about the data generating process for the return series under investigation (an insightful development of the theory and use of wavelets can be found in Percival and Walden, 2000; Gençay et al., 2001; Ramsey, 2002).

Let the $F(t) \in L^2(R)$ be a function (for $t = 1, \ldots, T$). The time dimensions can be expressed as a linear combination of a wavelet function

$$F(t) = \sum_k s_{j,k} \emptyset_{j,k}(t) + \sum_k d_{j,k}\, \psi_{j,k}(t) + \sum_k d_{i-1,k}\, \psi_{j-1,k}(t) + \cdots + \sum_k d_{1,k} \psi_{j-1,k}(t), \quad (1)$$

where the orthogonal basis functions $\emptyset_{j,k}$ and $\psi_{j,k}$ are defined as

$$\emptyset_{j,k} = \frac{\emptyset\left(\frac{t-\xi}{\xi}\right)}{\sqrt{\xi}} \text{ with } \int \emptyset(t)dt = 1, \quad (2)$$

$$\psi_{j,k} = \frac{\psi\left(\frac{t-\xi k}{\xi}\right)}{\sqrt{\xi}} \text{ with } \int \psi(t)dt = 0. \quad (3)$$

The scale or the width of these functions is denoted by $\xi = 2^j$, where j and k represent respectively the scale and translation parameters. Equation (2) presents the long-scale smooth components that are used to generate the scaling coefficients. Meanwhile, the differencing coefficients are generated the wavelets in Equation (3). The resulting multiscale decomposition in Equation (1) can be simplified as

$$F(t) = S_J + D_J + D_{J-1} + \cdots + D_j + \cdots + D_1\,, \quad j = 1, \ldots, J \quad (4)$$

where D_j is the jth level wavelet and S_J represents the aggregated sum of variations at each detail of the scale.

For the purpose of this study, the least asymmetric filter of length eight is taken to generate uncorrelated coefficients across scales. Further, the oscillation periods of 2–4, 4–8, 8–16, 16–32, 32–64 and 64–128 days that corresponds to wavelet scales D1, D2, D3, D4, D5 and D6 respectively are obtained. These scale and wavelet coefficients are estimated using the maximum overlap discrete wavelet transform (MODWT).

2.2 Copula-GARCH

Non-linear dependence in high dimensional data may be modelled independently of the marginal distributions. A copula is an interesting approach often taken to link univariate models as a function of other variables as well as of its own lagged values (Joe, 1997; Nelsen, 2003). In the financial econometrics literature, authors have combined the copula functionwith GARCH models to model financial data in which the marginal time series that follows a usual time-varying process and the dependence structure between them is specified by a copula function. Such a combination as copula-GARCH function offers an effective way of investigating the impact of certain joint stock-return realizations on the subsequent dependency of international markets (see, for example, Jondeau and Rockinger, 2002).

For the p-dimensional random vector (X) with $F_i(X_i)$ and H defined marginal and joint distribution, respectively, a unique t-copula function $C^t\colon [0,1]^\tau \rightarrow [0,1]$ for a multivariate t distribution with correlation matrix ρ and v degrees of freedom can be defined as

$$C^t\left(x_1, \ldots, x_p;\ \rho\right) = t_{p,v}\left(t_v^{-1}(x_1), \ldots, t_v^{-1}\left(x_p\right)\right), \quad (5)$$

$$C^t = \int_{-\infty}^{t^{-1}(x_1)} \dots \int_{-\infty}^{t^{-1}} \frac{\Gamma\left(\frac{v+n}{2}\right)}{\Gamma\left(\frac{v}{2}\right)(v\pi)^{\frac{n}{2}}\sqrt{|\rho|}} \left(1 + \frac{1}{v} Z^T \rho Z \right)^{-\left(\frac{v+n}{2}\right)} dZ_1 \dots dZ_n. \quad (6)$$

Among different pair-copula families, Clayton's is preferred for financial data since it allows for more asymmetric tail dependence in the negative tail than in the positive and is given by

$$C^{Clayton}(X_1, X_2) = \left(\max\{x_1^{\theta} + x_2^{\theta} - 1, 0\}\right)^{\frac{1}{\theta}}. \quad (7)$$

For $\theta \rightarrow \infty$ the Clayton copula implies comonotonicity, and for $\theta \rightarrow 0$ it implies independence (for more details see, among others, Nikoloulopoulos et al., 2012).

Having discussed the copula function, we are in a position to model the changes in the dependency structure between them. Suppose that the stock market return R_t can be written as

$$R_t = \mu + \varepsilon_t; \quad \varepsilon_t = Z_t\sqrt{h_t}. \quad (8)$$

Assume that each series Y_{it} follows the GJR GARCH (1,1) (see Glosten et al., 1993), represented by the expression

$$\zeta_t^2 = \delta + \alpha\varepsilon_t^2 + \gamma\varepsilon_{t-1}^2 d_{t-1} + \beta\, h_{t-1}, \quad (9)$$

where (α) and (β) measures the size effect and persistence of the shocks on volatility, while the sign effect is given by (γ). The impact of the shocks (news) is determined by the dummy such that $d_t = 1$ if $\varepsilon_t < 0$ (bad news) and $d_t = 1$ otherwise.

The marginal distribution function for each index is, therefore, defined as $F_i(X_i) = F_{\varepsilon_i}(\frac{Y_i}{\sqrt{h_i}})$. Further, the joint density, $f(Y_t)$, is then specified in terms of marginal distributions for the error terms, ϵ_t, combined with a copula function in Equation 5; that is

$$f(Y_1, \dots, Y_p) = C(x_1, \dots, x_p; \rho) \prod_1^p \frac{1}{\sqrt{h_i}} F_i(X_i). \quad (10)$$

The distribution function as well as the correlation matrix can then be obtained in the process of a model built by solving the maximum likelihood estimator of the parameter vector of each market return.

3. Data and Empirical Results

The data considered in this study are daily closing equity market price indices for six markets. In particular, we consider the Composite Index (S&P 500) for the United States, the S&P TSX Composite Index, (S&P/TSX) for Canada, the FTSE 100 Price Index (FTSE100) for the UK, the Nikkei 225 Stock Average Index (N225) for Japan, the Hang Seng index (HIS) for Hong Kong and the Shanghai Share

Index (SSE) for China. The HIS index enables us to investigate stock market contagion between Mainland China's markets and Honk Kong. Similarly, the S&P/TSX Composite Index is considered to investigate spill over effects in the North American region. Note that that the U.S. market is used as a numeral for the correlations. Therefore, below we consider the level of co-movement between the S&P500 and the stock markets listed above.

The sample covers the period from 1st January 2014 to 8th April 2020. Following the literature, stock returns are calculated as the difference between the logarithm of the price index. Further, the missing data arising from holidays and special events are bypassed by assuming them to equal the average of the recorded previous price and the next one.

As mentioned in the introduction, in the first step of the analyses, the maximal overlap discrete wavelet transforms (MODWT) is undertaken to obtain the multiscale decomposition of the return series under investigation. The return series are decomposed into different periodicity series, ranging from the shortest-periodicity series to the longest. In Figure 1-6, the WLMC maps are presented in a time-frequency domain on a scale by scale basis. For ease of interpretation, the left-hand horizontal axis is transformed to show the number of days in which the scale moves from low to high wavelengths. The heat maps indicate the increasing strength of the correlation among the stock markets indexes as they move from blue (lowest correlation) to red (highest correlation).

From Figures 1-6, there is clear evidence of long-run interdependence (at low frequency) between the U.S. stock market and the other markets before the start of the COVID-19 pandemic in December 2019. To be specific, starting with the correlation between the U.S. stock market and the U.K. market, Figure 1 indicates no sign of co-movement for the first 8-16 days, but correlation increases in the time scale D6 between January 2014 and June 2017. Similarly, in Figure 2, it appears that the U.S. and Japan stock markets have stronger long-term co-movement (at low frequency) since once again, we see the red colour in the D6 time scale. As for the correlation between the U.S. and China, weak correlation can be seen for the time scale D3 and below, as highlighted in Figure 3. Signs of contagion between the U.S. and Hong Kong stock markets can be observed in Figure 4, where shock events in the S&P 500 directly diffused to HIS. The correlation between the stock markets in U.S. and Canada, shown in Figure 5, indicates persistent co-movements between these financial markets since Canada has close commercial and financial ties to the US economy.

Once the impact of COVID-19 pandemic was felt worldwide, financial assets were immediately repriced. Panic spilled over all the major financial markets as indicated by the wavelet power of pairwise analysis analysed at lower scale brackets. Put differently, the co-movements (either positive or negative) seem to have been stronger during the COVID-19 pandemic in most of the series under consideration. Specifically, with the exception of Japan, the financial markets under consideration showed significant dominant signs of co-movement at periods of high frequency up to 2-4 days in length. In the case of the correlation UK and Canada, the market contagion appears to be even stronger, as indicated by the red colour in Figures 1 and 5.

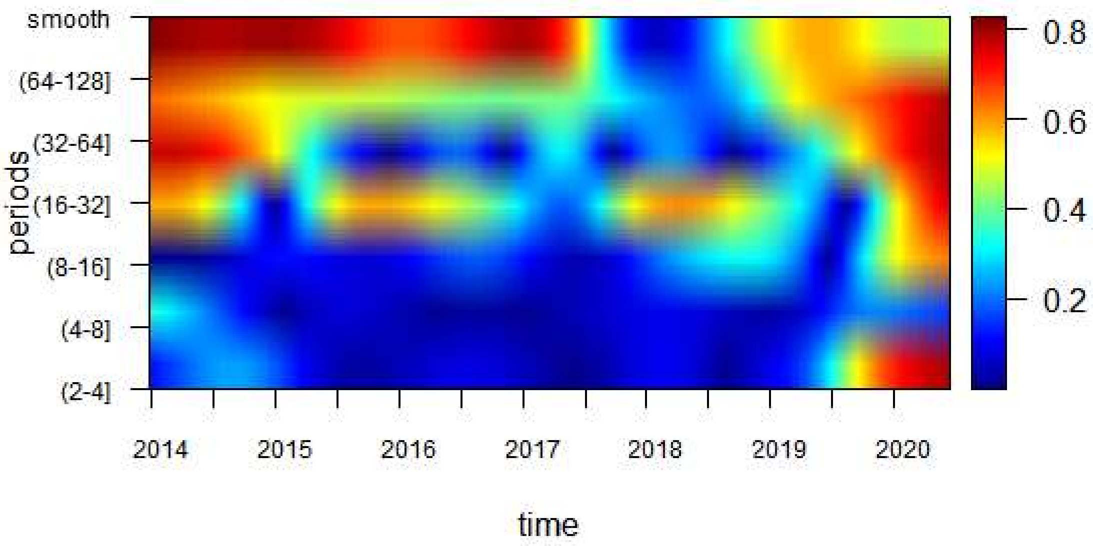

Figure 1: Wavelet multiple correlation between S&P500 and FT100 stock markets returns.

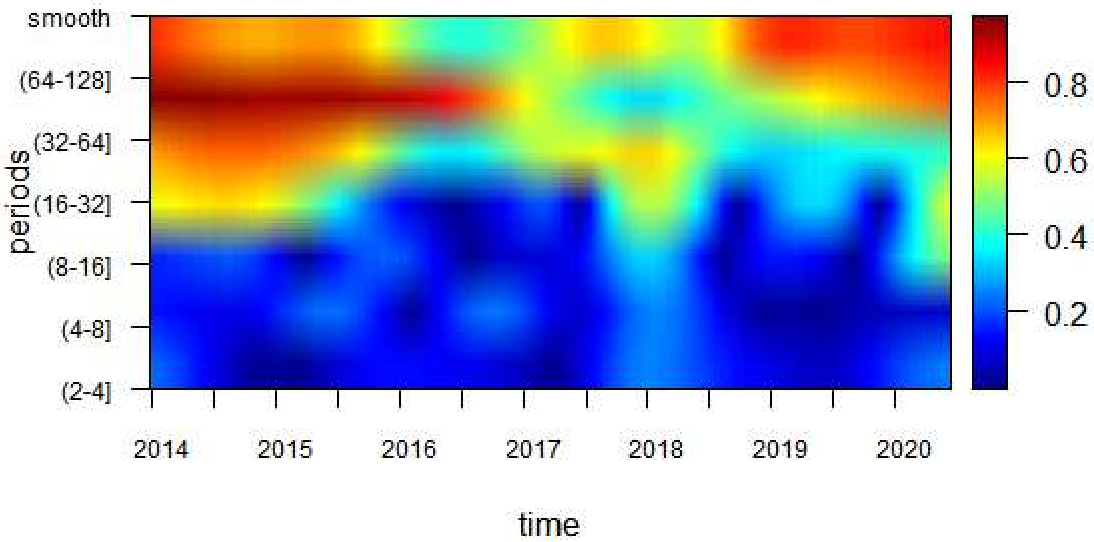

Figure 2: Wavelet multiple correlation between S&P500 and N225 stock markets returns.

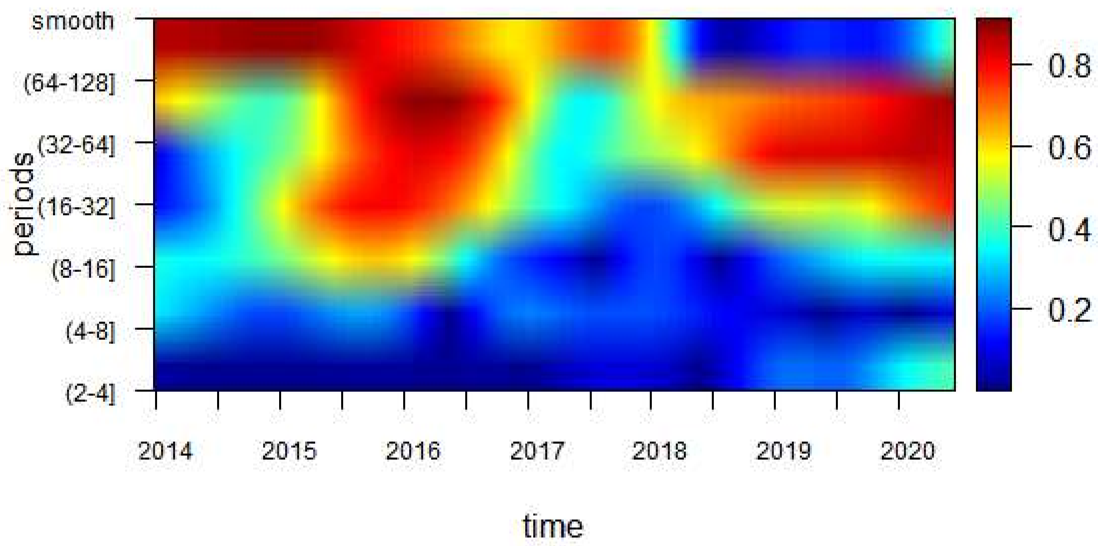

Figure 3: Wavelet multiple correlation between S&P500 and HIS stock markets returns.

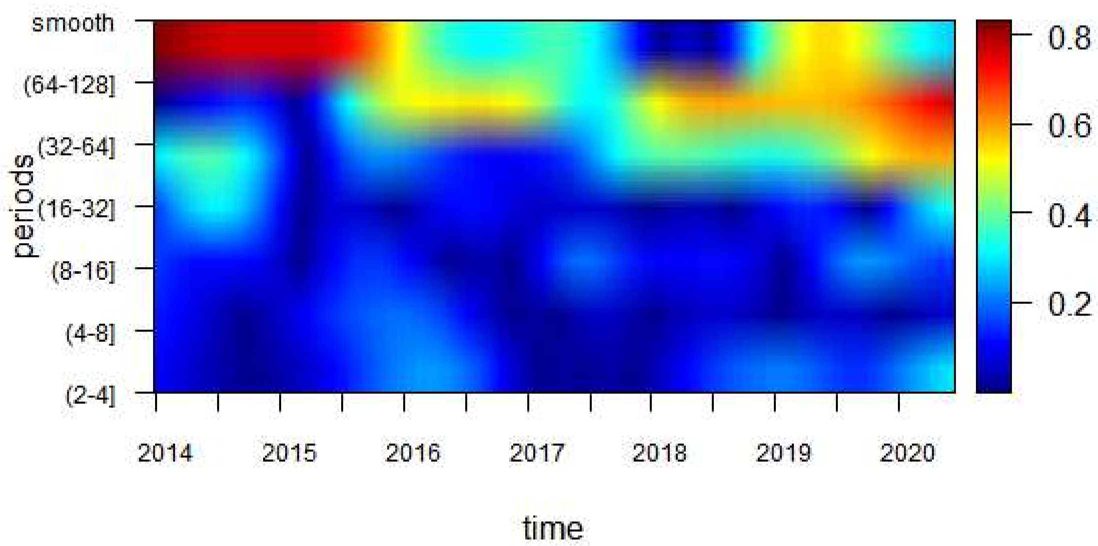

Figure 4: Wavelet multiple correlation between S&P500 and SSE stock markets returns.

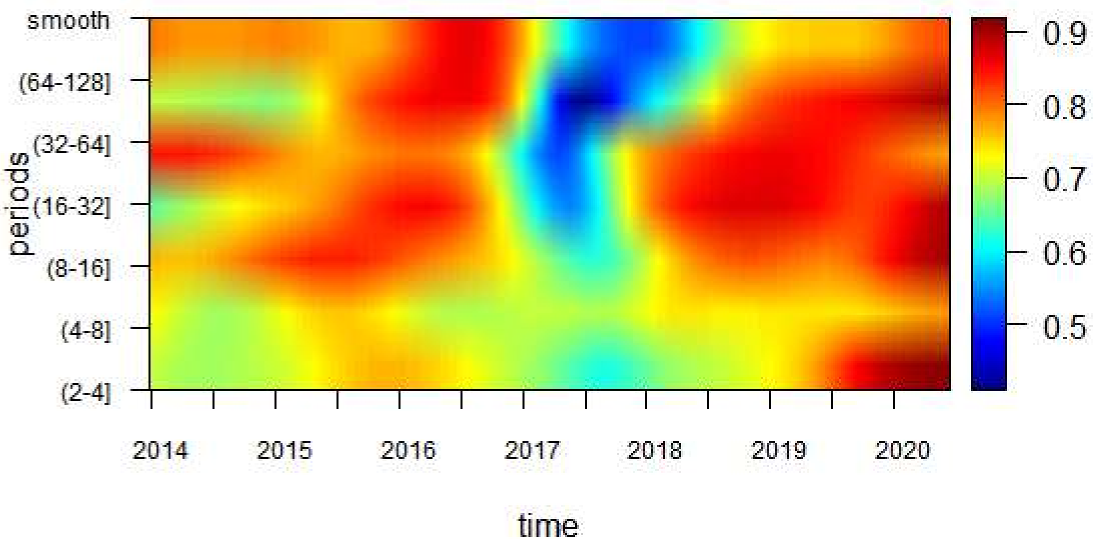

Figure 5: Wavelet multiple correlation between S&P500 and S&P/TSX stock markets returns.

Once the filtered series were extracted in the second step of our analysis, appropriate univariate GARCH model was estimated for the six stochastic processes at hand. Comparing a number of GARCH-type models, we concluded that the specification that best fitted the data under consideration was a GJR-GARCH model with a GHYP distribution for the innovation terms. [§] Table 1 reports the results of the copula-GARCH model for the pairwise dynamic correlation between the S&P500 index returns and the stock market returns for the other five countries considered in the scale frequencies D1-D6 as defined in Equation (4).

Table 1. Copula-Wavelets correlation between the S&P500 and other stock markets.

	D1	D2	D3	D4	D5	D6
FTSE100	-0.133	-0.157	-0.413	-0.570	-0.818	0.769
N225	-0.056	0.114	0.178	0.359	0.456	0.782
SSE	0.062	0.067	0.041	-0.187	0.269	0.414
HIS	-0.144	0.186	-0.331	-0.407	0.431	0.584
S&P/TSX	0.596	0.674	0.563	0.631	0.762	0.863

Note: the table reports the copula-wavelet correlations for oscillation periods 2-4, 4-8, 8-16, 16-32, 32-64 and 64-128 days defined as D1, D2, D3, D4, D5 and D6, respectively between the U.S. and the other stock markets under consideration.

[§] Note that the estimation results for the six GJR-GARCH models are not reported here, but they are available upon request.

In Table 1, it appears that the correlations substantially increase when the timescale increases. In this regard, from columns two, three and four it seems that the tail dependence is relatively weak in the short-run (time scales D1, D2 and D3) and increased by each decomposition in the pre-crisis period (as noted above). For time scale D4, in column five, it appears that the correlations are higher for all of the markets. In particular, the stock returns can be divided into closely correlated markets (Canada and the U.K.) with correlation coefficients around 0.63 and 0.57, respectively. Moderately correlated markets (Japan and Hong Kong) with correlation coefficients 0.36 and 0.41, respectively and mildly correlated markets for those markets whose correlation was less than 0.20, as in China for example.

From time scales D5 and D6, the differences in stock market interdependencies begin to show and are relatively high in D6. The U.K. and Canada have the highest correlation with the U.S., since the correlation is higher than 0.75 in these markets for the longest time brackets. Looking now at the remaining stock markets, also in this case the correlation increases with the time scale. For example, the correlation between the U.S. and Japan's was approximately 0.50 in D5 and increases to approximately 0.80 in D6, whereas Hong Kong's correlation between 0.411 and 0.584. China's correlation varied substantially and was eventually slightly lower than Japan's, at approximately 0.40 in D6.

Like the heatmaps in the WLMC map shown previously, these pictures changed during the COVID-19 pandemic. In detail, nearly all the assets reached a new level of asymmetric tail dependency up to scale 2 during this health crisis. The developed markets (UK and Canada) were the most closely correlated, with the values being close to 0.90. Similarly, markets in Hong Kong and China were approximately 0.40. Japan's correlation varied substantially and was eventually slightly lower than that of Chinese stock market.

Taken together, these results provide some support for the potential benefits of portfolio diversification. In this case, most of the markets (except for Canadian stock market) offered the best diversification benefits to a level of 4-8 days for U.S. short-term investors, since these markets generally have low correlations with the U.S. The benefits, however, disappear as the timescale begins to increase. As is evident from the table, the developed markets (the UK and Canada) follow the long-term trends of the U.S. more closely than the markets in other countries do. Nonetheless, China's correlation with the U.S. stays rather low for all of the tested timescales.

4. Conclusion

In this study we investigate the contagion effects between US stock market and five other major markets in the world focusing in particular on the COVID-19 pandemic outbreak between December 1919 and April 2020. Analysing the wavelet spectrum, we find evidence of long-run interdependence (at low frequency) for most of the stock market indexes under consideration before the start of the COVID-19 pandemic in December 2019. Moreover, our findings highlight that, before December 2019, the correlations were largely dynamic as the timescale increased. Subsequently, strong evidence of contagion was detected in nearly all the stock markets as the pandemic crisis drove them to new levels of asymmetric tail dependency.

Our findings highlight the advantage of using the time- and frequency-varying approaches as a tool for portfolio diversification during turmoil and calm periods. In our case, the diversification benefits for the developed stock markets vanished, but emerging markets, such as China's, still provide fertile ground for international diversification.

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