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Dipartimento
Economia e Statistica
Cognetti de Martiis

Working Paper Series

17/17

THE CREATIVE RESPONSE AND THE ENDOGENOUS DYNAMICS OF PECUNIARY KNOWLEDGE EXTERNALITIES: AN AGENT BASED SIMULATION MODEL

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THE CREATIVE RESPONSE AND THE ENDOGENOUS DYNAMICS OF PECUNIARY KNOWLEDGE EXTERNALITIES: AN AGENT BASED SIMULATION MODEL[1]

THIS WORKING PAPER INCLUDES THE APPENDIX

Forthcoming in JOURNAL OF ECONOMIC INTERACTION & COORDINATION

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ABSTRACT. The paper elaborates an agent based simulation model (ABM) to explore the endogenous long-term dynamics of knowledge externalities. ABMs, as a form of artificial cliometrics, allow the analysis of the effects of the reactivity of firms caught in out-of-equilibrium conditions conditional on the levels of endogenous knowledge externalities stemming from the levels of knowledge connectivity of the system. The simulation results confirm the powerful effects of endogenous knowledge externalities. At the micro-level, the reactions of firms caught in out-of-equilibrium conditions yield successful effects in the form of productivity enhancing innovations, only in the presence of high levels of knowledge connectivity and strong pecuniary knowledge externalities. At the meso-level, the introduction of innovations changes the structural characteristics of the system in terms of knowledge connectivity that affect the availability of knowledge externalities. Endogenous centrifugal and centripetal forces continually reshape the structure of the system and its knowledge connectivity. At the macro system level, an out-of-equilibrium process leads to a step-wise increase in productivity combined with non-linear patterns of output growth characterized by significant oscillations typical of the long waves in Schumpeterian business cycles.

Keywords: Creative reaction, Knowledge connectivity, Emergent property, Endogenous knowledge externalities.

1. Introduction

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[1] The authors acknowledge the financial support from the European Commission DG Research under Grant number 266959 for the 'Policy Incentives for the Creation of Knowledge: Methods and Evidence' (PICK-ME) project, as part of the Cooperation Programme/Theme 8/Socio-economic Sciences and Humanities (SSH), of the Collegio Carlo Alberto, and the University of Torino project IPER.

This paper contributes the literature that impinges upon the approach elaborated by Schumpeter (1947) according to which innovation is the result of the creative reaction of firms, facing unexpected changes in product and factor markets, contingent upon the availability of knowledge externalities. The availability of knowledge externalities, in turn, is the stochastic result of the introduction of innovations. Its persistence depends upon the actual amount of knowledge externalities that are generated at each point in time. This dynamics is the result of the interaction between individual decision making embedded in a system and the changing conditions of the system (Antonelli, 2011; Arthur, 2014).

The introduction of innovations requires the generation of technological knowledge. In turn, the generation and dissemination of technological knowledge can only take place in organized contexts characterized by appropriate levels of knowledge connectivity qualified in terms of viability of knowledge interactions and transactions among heterogeneous and creative agents that act intentionally to innovate when their individual performance is out of equilibrium. The generation of technological knowledge is, in fact, based on the interactive and collective recombination of internal and external knowledge through the intentional interaction and participation of a variety of learning agents embedded in a geographic and professional knowledge commons. Interaction is required for the acquisition and implementation of external knowledge, an essential input into the generation of new knowledge (Antonelli and David, 2016).

This process leads to the generation of knowledge stemming from internal research activities combined with knowledge externalities and strategic mobility across knowledge commons. The outcomes are determined by the structured contexts in which they are embedded, but they are also the cause of changes in the structure of the system, its knowledge connectivity and the pecuniary knowledge externalities available within the knowledge commons, likelihood of successful innovation and, thus, ultimately aggregate productivity. Innovation and changes to productivity levels affect the system’s price levels and the performance of firms, promoting new out-of-equilibrium conditions and new structures of the system (Antonelli, 2008, 2011, 2015a, 2016).

This open-ended feedback system is based on continual interactions between individual acts and endogenous knowledge externalities related to the structure of the system and its levels of knowledge connectivity. In this context, the decisions to both generate technological knowledge and introduce technological innovations by exploiting the knowledge interactions and organized structures in which they take place, are endogenous and are determined internally by the dynamics of the system. The individual and intentional actions of creative agents are central to the system dynamics; however, no single agent is solely responsible for or is able to forecast the eventual results of his or her actions because of the effects on the organization of the system (Miller and Page, 2007).

The characteristics of the landscape in which knowledge interactions and transactions take place play a central role in assessing the viability of knowledge generation strategies. Thus, feasibility of knowledge generation depends upon the knowledge connectivity of the system as measured by the levels of knowledge externalities, which, in turn, depend upon the characteristics of the knowledge landscape. These characteristics are neither static nor exogenous. They change continuously through time as a consequence of the activities of agents, and their capabilities to generate knowledge and introduce innovations and freedom to search for new opportunities for the generation of new technological knowledge. Changes to the features of the landscape engender both positive and negative externalities, which affect the capability of firms to innovate. The changing capabilities of firms to generate new technological knowledge affect their mobility and, ultimately, the contours of the space. Moreover knowledge landscapes and knowledge externalities are not given, but emanate from an endogenous, path dependent collective process that includes institutional changes such as the introduction of new intellectual property right regimes (Sorenson et al., 2006).

The present paper draws on the above to build a synthetic account of the role of externalities in the economics of technological knowledge, implementing the notion of endogenous knowledge externalities, showing the dynamic endogeneity of the emergence and decline of knowledge externalities at the system level, and exploring their implication for the rates of introduction of innovations and productivity increases in the system. Section 2 reviews the changing attitudes to knowledge externalities, and elaborates a theoretical framework to understand the endogenous dynamics of pecuniary knowledge externalities. Section 3 presents an agent-based model of the innovation system. Section 4 presents the results of the simulation focusing on the alternative hypotheses related to the institutional and architectural features of the innovation system. Section 5 concludes by summarizing the main results and discussing some policy implications of the analysis.

2. Knowledge Externalities as Input and Output of System Dynamics

Recent efforts to apply complex system analysis to the social sciences and to implement an economics of evolutionary complexity using agent based simulation models (ABM), are particularly helpful to analyse the generation of technological knowledge as an endogenous collective process that is both the key causal factor and the outcome of system dynamics. In this approach, technological knowledge and innovation constitute the emergent property of organized contexts characterized by qualified interactions among heterogeneous and creative agents able to re-act intentionally to innovate when their performance is out of equilibrium. The individual and intentional actions of creative agents are central to the system's dynamics, which are determined by the structure of the system and the endogenous dynamics of knowledge externalities. No individual agent can claim responsibility for or forecast the eventual results of its actions. The complexity of the system is promoted by the interdependence between individual action and structural change (Lane, 2002, Lane et al., 2009; Page, 2011).

Following the knowledge recombinant approach, in order to generate new knowledge, firms need to combine internal sources of knowledge, such as in house research and development (R&D) activities and learning processes, with the systematic (as opposed to the occasional, additive) use of external knowledge, which is acknowledged to be an indispensable input for the production of new knowledge. Its criticality, for the generation of recombinant knowledge to produce new technologies, forces learning agents to search for and access it intentionally. No firm can innovate in isolation. External and internal knowledge sources are substitutes only to a limited extent: complete substitution between internal and external knowledge is impossible. External and internal knowledge, both tacit and codified, are complementary inputs – neither can be dispensed with (David, 1993; Weitzman, 1996; Fleming, 2001; Fleming and Sorenson, 2001; Cowan and Jonard, 2004, Antonelli and Colombelli, 2015a and b).

The limited appropriability of knowledge engenders flows of knowledge spillovers. Their actual absorption and eventual use in the generation of new technological knowledge, however, is determined by the knowledge connectivity of the system. In turn the knowledge connectivity of the system is influenced by: i) the actions of learning agents that affect the structure of the system; ii) the knowledge interactions combined with internal learning efforts that affect the distribution of the knowledge possessed by each agent and made accessible through knowledge interactions. Similarly, mobility across the knowledge commons affects the density of agents and, hence, the amount of knowledge absorption costs.

Knowledge spillovers, in fact, do not automatically benefit all potential recipients (Griliches, 1979, 1992; Romer, 1990). Systematic and intentional efforts are required to exploit knowledge spillovers. This requires a knowledge exploration strategy to search, screen, identify knowledge sources and to assess whether and to what extent the firm can rely on that source combined with the stock of internal knowledge to produce new knowledge. The firm must be able to fully combine and coordinate the relevant learning and research activities conducted within its boundaries with the relevant sources of tacit and codified external knowledge, for the successful generation of new knowledge (Beaudry and Breschi, 2003; Bresnahan et al., 2001; Antonelli and Colombelli, 2015a and b).

Identifying and accessing external knowledge are expensive pursuits due to its direct purchasing costs, whether there are markets for the knowledge, and especially the costs of knowledge absorption. Knowledge interactions are required to access external knowledge - especially its tacit components - to reduce the risks to the vendor of opportunistic behaviour and knowledge leakage. It is difficult and costly to detail all the ingredients, necessary procedures, possible applications and implications of knowledge, and transfer of technological knowledge requires systematic codification efforts (Arrow, 1969; Mansfield et al., 1981; Lundvall, 1988). Knowledge is sticky; it is embedded in organizations, protocols and procedures. External knowledge acquisition and sharing can be achieved only via direct and purposeful interactions to create the appropriate institutional context, which entail specific costs. The capacity of agents to access external technological knowledge depends on the fabric of the relevant institutional relations, and on shared codes of understanding which help to reduce information asymmetries, limit the scope for opportunistic behaviour, and build a context that allows reciprocity and the building of trust and generative relationships (Antonelli and David, 2016). The receptivity of firms to knowledge generated elsewhere is not obvious. Its absorption requires dedicated activities that have a cost and vary across firms (Cohen and Levinthal, 1990, Antonelli, 2011).

The use of external knowledge as an input in the generation of new knowledge entails knowledge absorption costs related to: i) knowledge transactions, communication, and interaction costs associated with the exploration activities such as search, screening, processing, contracting, and interacting with competitors, suppliers and customers and ii) the processing costs associated with the access and actual use of external knowledge (Griffith et al., 2003; Guiso and Schivardi, 2007). In some specific locations heavy knowledge absorption costs make the access to external knowledge expensive. In others, knowledge absorption costs are low because of ease of access to the knowledge commons. These conditions are highly idiosyncratic and localized (Bischi et al. 2003; Zhang, 2003).

Pecuniary knowledge externalities are defined by the gap between the equilibrium cost of knowledge[2] as an input in knowledge generation, and its actual cost taking into account its limited appropriability and exhaustibility. Because of its limited appropriability knowledge cannot be fully appropriated and spills. Because of its limited exhaustibility it can be used again and again as an input in the generation of further knowledge. Its secondary use requires dedicated activities and hence absorption costs. The costs of the secondary use of knowledge *may* be –in appropriate circumstances and favourable conditions of knowledge governance within economics systems- lower than the equilibrium levels of knowledge as a standard good. Pecuniary knowledge externalities are defined by the gap between the costs of knowledge as a standard good and the actual cost of knowledge, taking into account its limited appropriability as well as its absorption costs.

The levels of the pecuniary knowledge externalities available within the knowledge commons, the resulting amount of knowledge that the overall system can generate, and the aggregate outcomes of the dynamics related to productivity levels are simultaneously endogenous and unpredictable, and subject to the changing interplay between individual action and structural change. In this approach, neither interactions nor the organized structures in which they take place are exogenous; they are determined internally by the system dynamics (Arthur et al., 1997; Lane et al., 2009; Antonelli 2011).

The levels of pecuniary knowledge externalities vary across commons and time. They depend on the density of the co-localized innovation agents in the region. The density of knowledge commons yields, in fact, both positive and negative effects on the actual levels of knowledge absorption costs and hence on the levels of pecuniary knowledge externalities. Density has negative effects on the amount of resources that are necessary to perform the exploration and search of external knowledge: the larger the density the more expensive the identification of the external knowledge items that are necessary to generate new knowledge. Density, however, has also positive effects

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[2] We define the equilibrium cost of knowledge as the cost of a standard divisible input, traded in a competitive market, that can be fully appropriated, wear and tear because has a clear exhaustibility, and is used to produce an output that is traded in a competitive market.

in terms of information processing. The larger is the density and the lower are the nunit costs of the commons within which each firm is located. Total knowledge absorption costs, as a consequence, decline with density until a minimum is reached. Beyond a threshold level of density, where knowledge absorption costs hit a minimum, knowledge absorption costs increase along with the density of the commons. The relationship between density and net knowledge pecuniary knowledge externalities exhibits the typical traits of a U-shaped functional form.

At each point in time, the actions of agents, including the generation of new knowledge and the introduction of innovations, affect the structure of the system, the architecture of networks, the density and quality of commons, the organization of communication flows and, ultimately, the determinants of external knowledge availability and its governance costs. Specifically, the mobility of agents in the regional space, related to accessing external knowledge available within a rich knowledge commons, has a direct effect on location costs as well as on the level of the knowledge governance costs. Both too little and too much density of agents can be detrimental to the accumulation and creation of firms' technological knowledge and innovation capabilities. This refers to the notion of endogenous knowledge externalities.

The characteristics of the system into which knowledge flows, matter in relation to the knowledge governance costs which include transaction, interaction, absorption and communication costs (Arrow, 1969). Because of the intrinsic non-exhaustibility and non-divisibility of knowledge, and its tacit and sticky characteristics, the costs of external knowledge, may differ from the long-run equilibrium cost defined by matching marginal costs with marginal production. This important U relation is strongly influenced by the level of the knowledge governance costs that reflect the characteristics of the structure of the system. Only if the costs of external knowledge are below the equilibrium level will firms react by innovating. The introduction of innovation is clearly an emergent property of the system, which occurs only in specific and positive geographic, institutional and sectoral contexts. However, the structural characteristics that yield net positive knowledge externalities and the resulting introduction of technological innovations, are local rather than global, are far from being static or exogenous and are determined by strong endogenous and localized dynamics (Krugman, 1994).

As a result, net positive knowledge externalities are a transient property of the system in which firms are embedded. Schumpeter (1942, (1950): 28) commented that: 'Surplus values may be impossible in perfect equilibrium, but can be ever present because that equilibrium is never allowed to establish itself'. The quality of the knowledge governance mechanisms in place is important when assessing the size of the net positive effects of knowledge externalities.

Pecuniary knowledge externalities are endogenous to the system in reflecting the changing distribution of co-localized members of the knowledge commons. They are inherently path dependent in stemming from elements of past dependence demonstrated by the stock of firms in the knowledge commons at each point in time, through the pervasive role of contingent factors such local interactions, feedback and strategic mobility of firms. The mobility of firms affects the net positive externalities available in each location. The entry of new firms is likely to increase the overall levels of knowledge governance costs and, the same time, may increase the opportunities for knowledge sharing. On the other hand, firms' exit indeed helps to reduce overall levels of knowledge governance costs but also affects the opportunities for knowledge sharing. The mobility of firms is fully endogenous; it arises from the search for better opportunities to generate new technological knowledge, promoted by out-of-equilibrium conditions. At the same time, firms' mobility, by changing the structural conditions of the system and its knowledge connectivity, affects the actual opportunities for generating new technological knowledge.

The ruggedness of the system in which firms are localized is not an exogenous characteristic –as it is assumed in NK models-, but is intrinsically endogenous and is determined by the firms' mobility.[3] The dynamics of the system feeds continuously on the interplay between out-of-equilibrium conditions, firms' reactions, enhanced learning processes, external knowledge search, mobility in the knowledge space, structural changes, a new balance based on knowledge externalities, the generation of new technological knowledge, introduction of productivity-enhancing technological innovations, price reductions and eventual new out-of-equilibrium conditions. Endogenous knowledge externalities are at the heart of the innovation system.

At each point in time, there may be several solutions, but each will be different in its standard characteristics of stability and replicability. Equilibrium points are erratic. Small shocks engendered by the mobility of firms seeking to absorb higher levels of external knowledge, have major effects at both the aggregate and disaggregate levels, and may push the system far beyond any given values although not backwards to levels experienced in a previous phase. The performance of individual agents and of the system at large, depends on the distribution within the system of agents across the knowledge commons, their density and their interactions, and their knowledge endowments. Each of these elements is interdependent with the others, and each stems from the dynamics of constantly changing collective dynamics.

Path dependence, because of the roles of learning and interdependence, exerts powerful effects. The stock of available knowledge and the systems of knowledge communication in place at each point in time, catch the effects of past dependence. However, small events can change the direction and affect the rates of these changes, so as to alter the trajectories set at the origin of the process (David, 2007).

3. An ABM exercise

3.1. The building blocks of the simulation model

ABM allows exploration of the workings of the interactions, transactions and feedbacks between individual actions and the system structure, which make up the simple, but articulated economic system outlined in the previous section. ABM provides a tool to grasp the dynamics of the complex interactions among agents, through the environment and between the environment and the agents within it, that arise from the simulation, i.e. the model computation, without the need for extensive

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[3] NK models assume the reverse, defining density of the components in the landscape, and their knowledge to be exogenous (Levinthal, 1997).

and detailed descriptions of the dynamics investigated. This approach models, in a parsimonious and simple way, the intrinsic complexity of the knowledge interactions that are allowed to affect the structure of the environment in which they take place (Axtell, 2005; Terna, 2009).

The ABM implemented in this section operationalizes, through the interactions among a large number of objects representing the agents in the system, the functioning of a typical complex process characterized by: a) a key role of knowledge externalities; b) augmented by the Schumpeterian notion of creative reaction conditional on the availability of knowledge externalities (Schumpeter, 1947; Antonelli, 2016); and c) enriched by the explicit assumption that the actions of agents affect the structure of the environment including the amounts of the pecuniary knowledge externalities (Lane, 2002, 2009 et al.)[4].

The model assumes bounded rationality of firms, and is based on appropriate criteria of conduct related to procedural rationality. Firms are endowed with the capabilities to learn and to react that enable procedural rationality augmented by the inclusion of potential creative reactivity. Firms are credited with the capability to try and react: their reactions are determined by the out-of-equilibrium conditions when profitability levels are far away from the average. Their reactions are creative and, when and if positive knowledge externalities are available, lead to the introduction of productivity enhancing innovations rather than only adaptations or adjustments between quantities and prices (Antonelli, 2008 and 2011).

In the ABM, demand and supply meet in the market place; production is decided ex ante, and firms try and sell their output in the product market, where customers spend their revenue. The match between demand and supply sets temporary prices that define the performance of firms. Firms are heterogeneous both with respect to their productivity levels and ultimate profitability, and with respect to their location. The economic system is represented as a collection of regions, or commons, across which firms are distributed at the start of the simulation process.

In the simulation, heterogeneous firms produce homogeneous products that are sold into a single market. In the product market, households expend the revenue derived from wages (including research fees) and the net profits of shareholders. In input markets, the derived demand from firms matches the supply of labour provided by workers, including researchers. For simplicity, no financial institutions are activated, and payments cannot be postponed. Firms' capital is supplied solely by shareholders, and all the commercial transactions are cleared immediately. Market clearing mechanisms based exclusively on prices maintain a perfect equilibrium between demand and supply. This equilibrium is ensured for both product and factor markets: quantities determine the correct price, enabling the whole production to be sold. No friction or waiting times are simulated, factors are assumed to be immediately available.

The production function is very simple and avoids issues related to different kinds of production processes, input availability, warehouse cycles and so on: outputs depend

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[4] See Antonelli and Ferraris (2011 and 2017) for complementary specifications of this model.

exclusively on the amount of employed labour and its productivity. Both labour and productivity vary among firms. Labour depends on the entrepreneur's decision about the growth of production; productivity is a function of the technological level achieved by the firm via innovation.

The whole output is sold in the single product market, where the revenue equals the sum of wages, dividends and research expenses, and the price depends on the liquidity. According to the temporary price levels, profits are computed as the difference between income and costs, no taxes are paid, and no part of the profit is retained by the enterprise. Shareholders either receive profits or reintegrate losses. Firms can support their losses only up to a certain threshold beyond which they leave the market and are replaced by new entries, after a parametric number of production cycles.

Firms are learning agents that are able to react to out-of-equilibrium conditions. According to their performance levels and the availability of external knowledge, firms can fund research activities dedicated to innovation. Firms learn internally by doing, and externally by interacting. Internal learning processes are intrinsic to the firm and occur spontaneously through time. External learning involves two aspects. First, the rate of internal learning is influenced by the local conditions of the commons. The accumulation of competences via the firm's learning processes is greater, the greater the average productivity of all the other competitors co-localized in the commons. Second, we assume that localization in a knowledge commons provides the opportunity to absorb technological knowledge from co-localized firms with higher levels of productivity. External learning entails specific knowledge governance costs required to carry out the necessary activities of knowledge networking and communication among all the members of the commons. Knowledge governance costs depend on the number of firms within each commons by means of both fixed and variable costs. Fixed costs stem from the administration of the common: the level increases with the size of the common, but unit costs for each firm decline as the fixed costs are shared with the other members of the commons, independently of the need and opportunity for external learning. Next to fixed costs there is the variable part of the knowledge absorption cost that is proportional to the number of firms in the common. In this way the cost function that relates the amount each firm has to bear to be part and take advantage of a commons, to the population of the commons, becomes a U shaped curve.

The whole system is represented as nested collection of agents; agents are grouped in commons that are constituted by a simple collection of agents; the collection of commons constitutes the whole system (a collection of collections of agents). The simulation process shows that the localization of the agents in different commons is the result of their past activities although these can change at each point in time. The results from a production and consumption cycle influence the strategies adopted by the agents during the next cycle. Hence, the dynamics of the model is typically characterized by path dependence: the model dynamics is non-ergodic because history matters, and irreversibility limits and qualifies the alternative options at each point in time. However, at each point in time, the effects of the initial conditions may be balanced by occasional events that could alter the 'path', that is, the direction and the pace of the dynamics (David, 2007).

Firms perform basic search functions and acquire information about the levels of profitability of neighbouring firms in the same commons. As a result of bounded rationality, the firms in the model are not able to observe the entire economic system, but only the average levels of profitability of the other firms. Individual transparency is clearly local: the spectrum within which firms can observe the conduct of other firms is limited to the particular commons.

The farther profitability lays outside the local average, the stronger the out-of-equilibrium conditions. If profitability results are below average, firms can innovate in order to improve their performance; when results are above average, they can take advantage of abundant liquidity and reduce the opportunity costs of risky undertakings. Innovation is viewed as the possible result of intentional decision-making that takes place in out-of-equilibrium conditions. The farther the firm from equilibrium the more likely that it will innovate. Hence, we assume a U-shaped relationship between levels of profitability and innovative activity, measured by rates of increase of total factor productivity.

To summarize, the firm's motivation to innovate increases each time its performance is found to be far enough from the local average. The motivation becomes progressively stronger if the enterprise's relative position remains outside the band for several and consecutive production cycles: after a parametrically set number of consecutive cycles the enterprise performs an innovation trial.

Out-of-equilibrium conditions push firms to try to react by generating technological innovations that will increase their productivity. Attempts to generate new technological knowledge and to innovate are based on internal research and learning efforts, and access to external knowledge available within and across commons. Search for and access to external knowledge can be both local and global. When the neighbourhood in which each firm is embedded does not provide sufficient opportunities to generate additional technological knowledge, firms can move within knowledge space across commons, to get closer to firms with high levels of technological knowledge. The absorption of external knowledge requires dedicated resources and specific costs, as does mobility across commons to achieve proximity to firms with higher levels of productivity.

Building on the growing empirical evidence on the intrinsic characteristics of agents' dynamics, we characterize the search activities at the base of the innovation process in our learning firms, as typically displaying *Levy flight* traits. We suppose that firms alternate extended phases of local search within their own commons with long jumps that take them to other commons (Barabasi, 2010). Hence, we assume that the generation of additional technological knowledge takes place when the learning firm is able to master a three-step sequence consisting of: i) valorization of internal competence based on learning processes; ii) local (within commons) absorption of external knowledge; and iii) entry into a new commons characterized by higher levels of net pecuniary knowledge externalities.

The successful generation of new technological knowledge at the same time yields new knowledge externalities and enables the introduction of productivity enhancing innovations. Their introduction, in turn, reduces the overall price levels in the product markets, (affects the working of factor markets) and creates new out-of-equilibrium conditions. The micro-macro dynamics loop is closed, and engenders continuous growth and change provided that changes to the system structure–do not promote provision of positive net knowledge externalities. The interaction between individual action and systemic change includes the new knowledge externalities that spill from the limited appropriability of the new knowledge and the structural changes determined by the mobility of firms across the knowledge commons, and its effects on knowledge governance costs. Endogenous knowledge externalities are the engine of system dynamics. Their level is not given and static: it can increase and decrease according to the amount of innovations being introduced at each point in time and hence the amount of knowledge generated at each point in time taking into account the changing levels of knowledge connectivity determined at each point in time by the changing structural landscape of the system (Anderson et al., 1988; Rosser, 2004).

3.2 A detailed presentation of the innovation process simulation

Since the paper aims to identify the changing role of endogenous knowledge externalities in the innovation process, here we explore the ABM of the innovation process in detail, and stress analytically the role of the external factors that shape the recombinant generation of technological knowledge. The Appendix provides a detailed presentation of the basic components of the analytical model and the simulation parameters.[5]

Firms are characterized as learning agents. Learning is both internal and external to the firm:

i) internal learning is a routine that includes typical processes of learning by doing and learning by using. Internal learning enables the accumulation of tacit knowledge and potentially competence that requires a specific action to be eventually mobilized and transformed into concrete technological knowledge. External learning processes influence the rates of accumulation of each firm;

ii) external learning is also a routine and consists of monitoring activity that enables firms to assess the profitability levels and productivity levels of the other firms co-localized within the commons. External learning relies on interactions with other firms in the same commons. Bounded rationality confines firms to observing only other firms in their particular commons. External learning provides information on the availability of external knowledge that can be tapped if and when the firm tries to upgrade its productivity level. External learning encompasses two processes: i) faster learning rates, influenced by the average productivity of the commons; and ii) the possibility to absorb technological knowledge from co-localized firms with higher productivity levels.

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[5] A set of Appendices is available on request. Specifically Appendix A for a detailed presentation of the pseudo code of the model, Appendix B for the parameters of the model, Appendix C for the parameters of the simulation.

Agents follow a satisficing approach in their decision to try and innovate. At each point in time, learning firms assess their own profitability against that of co-localized firms within the commons. If their profitability is either below or above the local average, the firm will react. Their reaction may be adaptive or creative according to the availability of knowledge at a cost that is below the marginal product: innovation efforts are expensive because innovation is not free. Firms are short-sighted and can expend, in one unit of time, all their innovation budget including absorption costs even when the productivity gains obtained from absorption extend over more than one (1) unit of time. Innovation efforts can fail if the innovation costs exceed the productivity gains. In this case the reaction of agents will be adaptive. This takes place when the knowledge connectivity of the system is small and the levels of knowledge externalities are low. If knowledge is available at costs below its marginal product, the innovation efforts may be successful resulting in a creative reaction.

The innovation process consists of three sequential phases. In the first, firms try to mobilize their internal slack competence. In the second, firms with insufficient potential competence based on past learning processes, will try to absorb external technological knowledge spillovers from within-commons neighbours; if this is not possible, the third phase consists of a random move to another location in a different commons. Let us consider each of these in turn:

a) Firms consider the possibility to change their production technology when their performances are out-of-equilibrium and differ from the average. Out-of-equilibrium conditions are the result of mismatches between expected and actual product and input markets conditions. Firms in out-of-equilibrium conditions try and innovate. To innovate firms mobilize internal slack competence accumulated through learning processes and access external knowledge. The firms in our model are endowed with the ability to improve their production cycles. With each production cycle, the firm acquires and cumulates some technological potential. This potential requires intentional and dedicated research activities for its transformation into innovation. Competence can be transformed into innovation at a cost. Internal slack competence however is not sufficient to support the recombinant generation of new technological knowledge and the introduction of a productivity enhancing innovation: external knowledge is an indispensable, complementary input. In order to access and use external knowledge firms will try to access and absorb knowledge spilling from other firms. The search for external knowledge takes place locally within their own commons and at distance in neighbouring commons.

b) Local absorption enables exploitation of technology introduced by other firms. Firms can take advantage of their information acquisition from external learning processes, and can identify more profitable, co-localized firms. Absorption requires dedicated activities; and due to absorption costs, it is not free. Effective access to external technological knowledge requires substantial resources for exploration, identification, decodification and integration into the internal knowledge base. The absorption of knowledge from firms with higher levels of productivity is neither free nor unlimited. First, absorption of external knowledge requires specific activities and resources that have a cost. The level of these costs depends on the productivity gap between knowledge recipient and possessor. Second, the knowledge connectivity of the system plays a major role. When knowledge absorption gives poor or null results, firms move to another location in order to better address their technological conditions.

c) Mobility across commons. The third way to improve productivity levels involves moving around the physical space in order to identify more interesting commons. When mobilization of competences and within-commons knowledge absorption are not viable solutions, firms can try to move randomly to another location in the hope of finding superior knowledge, and a higher stochastic possibility to absorb technological knowledge from firms with high productivity levels. Since firms have access to individual information only about firms in their own commons and not all the other firms in the system, the Levy flight is blind. This random move can lead to superior as well as inferior commons. Thus, firms decide to move only if the profitability of their commons is below the system average. If it is above the average, the chances of finding a superior commons will be low. The conduct of firm shapes the structure of the system and, at the same time, the structure of the system influences the innovation chances of firms in several ways. Localization in an advanced commons is beneficial because: i) learning is faster, and ii) prospective recipients have higher possibilities to observe and absorb technological knowledge that high-productivity firms cannot fully appropriate; however, at the same time, iii) localization in a dense commons engenders high costs of search and interaction with the possible reduction of net pecuniary knowledge externalities.

3.3 The analytical representation of the simulation model

This section presents the analytical organization of the simulation model and the founding equations[6]. The production activity is specified following a simple linear function:

1) $O_i = A_i Lp_i$.

Where the output (O), of a generic i-th enterprise, depends upon the labour employed in the production cycle (Lp) and its productivity (A). The latter can vary between 0 and +∞. Customers (i.e. workers, share holders and researchers) spend the whole amount they earn in buying goods, so the selling price for goods is simply computed as:

2) $p = Y/\sum O_{i.}$

Where Y represents the whole amount earned by the customers and the sum compute

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[6] See Antonelli and Ferraris (2011 and 2017) for other complementary specifications of this simulation model.

the total production of enterprises operating into the simulated economy. The amount of wages represents the whole costs of the enterprises: research costs, as well as moving ones and costs related to the exploration of the common (that depend on the size of the common) are simply computed as work unit to be bought.

The amount of work units the enterprises demand for each cycle is determined as:

3) $L_i = Lp_i + CC_i + T_i + M_i$.

Where T_i represents the work units required to transform accumulated knowledge in technological innovation (either for internal learning or spillover from other firms into the commons) CC_i measures the work units needed to access the common knowledge based they include the research costs to increase the technological level by means of the use of external knowledge spilling in the commons where each firm is located (external learning) and M_i represent the work units needed to perform a movement from a common to another one (mobility across commons). Note that Lp_i represents the whole input for a firms, in this way the whole stylized economy becomes quite simple.

The unit wage (w) for a single work unit is the same for each enterprise; it is centrally computed as a constant value equal to one, under the assumption of an unlimited supply of labour:

4) $w = 1$.

Each firm pays its workers a total amount of wages (W) of:

5) $W_i = wLp_{i.}$

The whole amount of wages is simply computable as:

6) $W = \sum W_{i.}$

Firms decide to try and change their technology when their performances differ from the average in both cases of profit or losses. The resources invested to try and change their technology are defined whereas in the former case by the amount of extra-profit (the levels of extraprofits will be the maximum affordable investment), in the latter such amount is measured by the savings the enterprise realizes by reducing its input (labor) acquisition. In this way the adaptive response of enterprises is driven by profits: with a loss they reduce the amount of factors demanded and viceversa when they enjoy profits, whereas the reactive response is driven by the difference between the results of each single firm and the average results of the firms into the common each of them belongs. The amount a firm invests, both in case of internal learning or spillover, is computed as:

(7) $I_{i1} = \min(T_{i0}, \text{profit}_{i0}) \mid \text{profit}_{i0} > \text{tolerance};$
or as:

(8) $I_{i1} = \min(T_{i0}, (-\Delta L_{i1} * \text{labor price})) \mid \text{profit}_{i0} < -\text{tolerance}.$

For enterprises that perform moving strategies equation 7 and 8 work as well by simply substitute M_{i0} instead of T_{i0}.

Note that one action only can be taken in each cycle. Firms invest their resources in three ways: i) to transform their accumulated competence and to access external knowledge, ii) to transform spilled over technologies obtained by exploiting the information retrieved by belonging to a common organization, iii) to move to another commons in the hope – the flight is blind – to find there better conditions. Let us analyse them both in detail.

The transformation of their accumulated knowledge in new technology (so called internal learning) can be performed only if the firm has accumulated a minimum amount of knowledge specified through the parameter “productivityUpgrade” and could be performed for every amount greater than this amount at a time. The cost of the process is fixed to the value, in unit of work, specified for the parameter “transformationCost”:

9) $T_i = \text{transformationCost} * \text{internalLearning} / \text{productivityUpgrade} \mid \text{internalLearning} > \text{productivityUpgrade}.$

To access external knowledge, firms search in the knowledge commons and bear the knowledge absorption costs (CC_i) that are related to the size of the common and included into the commons costs each enterprise have to pay to be part of it. The relationship between density and knowledge absorption costs is U shaped. For low levels of density, the larger is the density of the common and the lower are knowledge absorption costs. Beyond a threshold, after the minimum, knowledge absorption costs are larger the larger is density: the costs occurred to access and process information about the knowledge spilling from the other firms increase with the density. These types of costs are computed, each cycle, in work units and have the same amount for each enterprise.

The amount of knowledge absorption costs (CC) is parametrically determined in each simulation through the parameter “commonsCost”, and depends upon the number of firms into the common (N). First the fixed component of the cost is computed as commonCost times the theoretical maximum number of components, i. e. the number of agents that populate the whole economy (N), this spreads evenly among the firms belonging to each commons (n_i). The variable part of the knowledge absorption cost is proportional to the effective number of firms that belong to the common (n_i). The following formula resumes the costs each component of the i-th common has to bear to belong to it:

10) $cost_i = (commonCost * N) / n_i + commonCost * n_i$.

The amount of external knowledge each firm can access depends upon its distance from the "spilling" firm. This distance (delta) is computed as follows:

11) $delta = (A_j - A_i) / A_j$

Where i is the enterprise that try and access external knowledge spilling from firm j. Note that it is possible to take advantage only of technologies whose patent license is expired. In order to transform the spilled over technologies the firm has to bear the transformation cost, in the same measure it has to bear to transform internal learning. To tackle the spillover each firm has to invest an amount in working unit that is:

12) $T_i = transformationCost * (A_j - A_i) / productivityUpgrade$.

The actual access to external knowledge takes place with a probability defined spilloverMinProb. The probability the spillover was successful (Ss) is:

13) $Ss = (1 - spilloverMinProb) * (1 - delta) + spilloverMinProb$

If no local knowledge pecuniary externalities are available because: i) spillover is not allowed, or ii) no firm provides suitable spillover in the commons (included the special case when the firm is the unique firm of the commons), firms try to move to another commons. This activity has a fixed cost set to the value, in units of work, specified for the parameter "movingCost":

14) $M_i = movingCost$

The outcome of their move will be positive so as to fuel a creative reaction and introduce innovations when and if the cost of knowledge –after taking into account knowledge absorption costs and moving costs- is below equilibrium levels. The dynamics of the system is now fully set. Firms caught in out-of-equilibrium conditions, with performances that are below or above the average, try and react by means of the introduction of innovations. In order to introduce innovations they try and take advantage of pecuniary knowledge externalities. To do so they may move from a knowledge commons to another. Their entry and exit affects the amount of pecuniary knowledge externalities available in each commons[7].

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[7] Note that the system is analytically consistent. Naming Π the profit of a generic enterprise and D the dividend it will pay to its shareholders, and remembering equations 1, 2 and 5 it is possible to write the following equations:

15) $D_i = \Pi_i = pO_i - W_i$

3.4 The **system dynamics of endogenous knowledge externalities**

Let us summarize the key points of the ABM to stress the relevance of endogenous knowledge externalities for the system dynamics. Appreciation of the endogeneity of knowledge externalities captures the characteristics of endogenous growth shaped by the intrinsic path dependent dynamics of the system at both the structural and macroeconomic levels.

At the start of the simulation, heterogeneous firms, localized in different commons, are endowed with different levels of productivity that are randomly distributed in the range]0,0.25[following a uniform probability distribution. Firms start the production process at their particular productivity level, try to sell their goods on the product market, and experience different levels of profitability. They compare their profitability with the average in the commons to which they belong. If their profitability is either below or above the local average in their commons, these firms will try to change their knowledge base and introduce technological innovations. These innovation efforts are deemed successful if their costs are below the value of their gains in terms of productivity in one unit of time. The costs of knowledge have a major influence on assessing the viability of innovation efforts.

Innovation efforts consist of a sequence that starts with the valorisation of their internal competences based on internal learning processes influenced by local average productivity levels. If the internal competence is not sufficient to introduce a new technology in order to increase productivity, firms move on to the second step and build on the information gathered through knowledge governance activities, to try to absorb knowledge from co-localized (within the same common) firms with higher profitability. If no such firms exist locally, then they move to the third step and attempt to move out of the original commons. Bounded rationality prevents assessment of whether the level of the knowledge governance costs in the new commons is lower than the advantages stemming from the external knowledge. The leap is blind. In the case of a negative outcome, the firm will continue to move across the system, to other commons.

This mobility of firms has important consequences for the system's structural landscape and the endogenous generation of knowledge externalities. Location in a knowledge commons is expensive due to the knowledge governance costs entailed in the resources required for searching, screening and assessing the levels of knowledge

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Where D could be less than zero if a loss had to be reintegrated. The amount of dividends paid to the whole systems is:

16) $D = \sum D_i$.

At the aggregate level the system could be resumed as follows:

17) $Y = \sum W_i + \sum D_i$.

By specifying D_i using equation (16) it is possible to obtain:

18) $Y = \sum W_i + \sum pO_i - \sum W_i$.

By operating simple compensations equation (18) becomes:

19) $Y = \sum pO_i$.

Recalling expression 2) it is evident that the whole system can reach equilibrium and the amount of money into the system remains always constant.

of the neighbours, and the costs involved in activating communication channels and networking interactions with them. The density of firms in a knowledge commons determines the level of knowledge governance costs with the result that the mobility of firms across commons affects the knowledge governance costs of all other commons members. Firm exits impact on knowledge governance costs too. Entry and exit impact may be either positive or negative depending on the number of firms belonging to the common, i.e. to the current position of the common on the U shaped cost curve. The levels of net pecuniary knowledge externalities available in a knowledge commons are strictly endogenous to the local system, with important dynamic effects.

The distribution in space of agents, scattered randomly at the beginning of the process, becomes fully endogenous as agents move across knowledge commons in the regional space, in the search for access to external knowledge, from the spillovers of proximate high-productivity firms. At the same time, since pecuniary knowledge externalities are endogenous, the actual level of net positive pecuniary knowledge externalities available at each point in time, within each knowledge commons, change over time as a consequence of the mobility of learning agents, and the consequences -in terms of knowledge governance costs- for all the members of the knowledge commons.

Hence, the dynamics of the regional distribution of agents exhibits traits typical of path dependence. The process is non-ergodic, but not past-dependent: small variations may exert important effects in terms of emergence of a strong commons or determine its decline and force firms to exit with their progressive dissemination in space. At the system level, excess entry in a ‘fertile’ knowledge commons may halt the generation of new technological knowledge and affect the rate of increase of productivity: excess knowledge governance costs reduce net positive pecuniary knowledge externalities to zero. This is most likely in commons populated by high-productivity firms since their higher levels of technological knowledge are likely to benefit firms that are willing to innovate and having casually landed to such commons will enjoy the possibility to exploit their new position.

The introduction of productivity enhancing innovations affects the position of the supply curve and modifies the conditions of the product markets: prices as well as the profitability of all incumbents will fall. Firms will re-assess their profitability levels with respect to the local average, and the process will keep going provided that changes to the structural conditions of the system promoted by the mobility of firms in the space, have not engendered the provision of knowledge externalities. The mobility of firms is the prime internal factor in the endogenous dynamics of the landscape and, hence, in the endogenous determination of the levels of knowledge externalities that shape the viability of the innovation process at firm level (Antonelli, 2011).

This loop affects the system in four ways. Specifically we expect to see:

i) at firm level, the levels of endogenous knowledge externalities may inhibit or foster the successful introduction of innovation;

ii) at the structural level, the dynamics exerted by the interplay between centrifugal and centripetal forces changes the structure of the system and the attractiveness of different commons. When knowledge governance costs exceed the benefits from external knowledge, centrifugal forces are at work: the density of commons declines with the exit of firms. Centripetal forces are at work when the benefits of external knowledge are greater than the sum of the knowledge governance costs: the size and density of the commons increases. The structure of the system is characterized by changing heterogeneous ‘stains’, indicating commons where the introduction of productivity-enhancing innovations takes place and commons where no innovation is possible. The distribution of these ‘stains’ changes continuously over time;

iii) at the commons level, the dynamics of output and productivity is characterized by typical Schumpeterian waves as the changing interplay between centrifugal and centripetal forces engenders different phases that affect the overall, aggregate rates of productivity and output growth which exhibit both growth and decline;

iv) at the macro-system level the dynamics of the system is likely to exhibit a step-wise process of output and productivity growth. The wave-like change at commons level in aggregate engenders a positive outcome, with phases of fast growth shaped by the upsides determined by the prevalence of centripetal forces, and phases of slow growth where the downsides are due to the stronger impact of centrifugal forces.

4. Results[8]

The results of the simulation confirm that the model is consistent, and is able to mimic the workings of a complex system based upon a large number of heterogeneous agents - both on the demand and the supply side- that are price takers in product markets where they are able to make efforts to react to changing market conditions. Replication of the temporary equilibrium price in the long term confirms that the model is appropriate to explore the general features of the system when the reaction of firms is adaptive and consists only of price to quantity adjustments. In the extreme case where firms cannot innovate due to lack of internal competence to mobilize, and lack of external knowledge to be absorbed, the system effectively mimics a static general equilibrium in conditions of allocative and productive efficiency, with no dynamic efficiency. The markets sort out the least-performing firms and drive prices down to the minimum production costs. This result is important because it confirms static general equilibrium as a simple and elementary form of complexity that emerges when agents are unable to innovate. As soon positive levels of knowledge externalities allow agents to react successfully to changing market conditions, by innovating, the equilibrium conditions turn dynamic and key system elements, such as price, quantities, efficiency and structure, keep

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[8] See the Appendix for the robustness and sensitivity checks.

changing (Antonelli, 2011; 2016). The dynamics, however, is not steady: the action of firms may engender negative effects on the knowledge connectivity of the system that in turn reduces the levels of net pecuniary knowledge externalities.

The results of the simulations of the model confirm the crucial role of endogenous knowledge externalities: with no positive externalities, productivity growth is much lower compared to when externalities are at work. The dynamics of the simulated system exhibit a wave-shaped trend describing firms continuous search for more profitable commons. These results were achieved using a plausible but not fully calibrated parameter configuration and, thus, need to be confirmed by a deeper investigation.

The simulation results confirm the existence of different areas within an economic system, where productivity grows at different rates, and profits follow different distributions over time as an outcome of the endogenous effects of each firm's relocation decision. In this process, commons are continuously augmented and reduced: new firms arrive, and existing firms move to other commons, with the balance between incoming and leaving agents mostly unable to maintain the commons population stable. Thus, their size is varying with each simulation step.

Depending on the capability of commons to retain agents, a single commons could operate as an attractor dramatically expanding its size. As already mentioned, since the Levi-flight is blind, agents move randomly to a new commons, but do not move if their profits are close to the average profit at the macro commons level, or their commons profitability is greater than the average profitability of the whole economy. The more a commons grows the more the knowledge governance costs for firms increase. When the costs overcome the benefits due to net positive knowledge externalities, firm profits start to fall inducing them to relocate to try and find more profitable commons.

Simulations demonstrate that the distribution of firms and, consequently the actual levels of net positive knowledge externalities, are the product of an endogenous process. Starting from a uniform distribution of firms across ten commons, the continuous relocation of agents produces a sequence of growth and decay of the commons according to the level of net positive pecuniary knowledge externalities their aggregation is able to engender.

The high technological and productivity levels achieved by more developed commons tend to become diffused as firms in these commons decide to move to less developed locations. Average productivity levels are very similar among commons because, in less developed commons, the higher knowledge brought by new entries from more developed commons rapidly spills over due to centrifugal forces. The decay of a former extensive commons is the means of sharing the effect of knowledge externalities with other commons, and provides valuable opportunities for less developed firms to make the leap towards higher productivity.

Specific simulations have been done to focus a number of key issues such as the existence and effectiveness of positive externalities. The findings come from comparing the results for four scenarios differing in the intensity of externalities: i) Alpha represents the benchmark scenario with full deployment of both types of knowledge externalities: internal learning enhanced by the average productivity of the commons, and opportunities to absorb external knowledge at low knowledge governance costs; ii) Beta excludes knowledge governance costs and enhanced internal learning, but includes the cheap absorption of external knowledge; iii) Gamma excludes knowledge governance costs, but includes internal learning at a fixed rate based on accumulation of experience, and independent of the average productivity of the commons; iv) Iota excludes knowledge governance costs and allows only internal learning at a fixed rate based on accumulated experience.

1. Compared dynamics of the benchmark scenario, Alpha, where the accumulation of experience proceeds at a faster pace in more developed commons but knowledge governance costs grows more than proportionally than population, to other ones.
2. Dynamics with different number of commons (Theta scenario).

In order to enable full comparability of the results, all the simulations in the second group were computed using very similar parameter set ups (few values change among the different scenarios), the same number of agents, same duration, same number of commons and same random distribution. Specifically, each scenario simulation is run for 2,000 production cycles, involving 1,000 agents. Scenarios Alpha, Beta, Gamma and Iota used ten commons, while in scenario Theta agents are grouped in only four commons because this scenario studies the influence of a different dispersion of agents.

At the onset of the simulation, levels of productivity are scattered randomly for each firm between 0 and 0.25, following a uniform random distribution, firms are endowed with initial accumulated knowledge randomly distributed between zero and 0.1 - the minimum knowledge level that can be transformed into increased productivity.

Information flows among agents are allowed only within each commons, where agents are able potentially to observe at each moment, all the other agents in that commons even when their number becomes quite large. Agents have no information on other commons, but do know the average profitability of the whole economy (macro system level), and that of the commons they belong to (macro commons level).

When an agent’s cumulated losses exceed a parametrically fixed threshold, the agent exits the market and goes out of business. After few cycles (another parameter) it is replaced by another agent endowed with technology equal to the average level in the commons. In order to exclude results were simply due to random events, simulations sub 1 and 3 have been run one hundred times by varying the random seed – used to set up pseudo random distributions – and their results are presented as average figures of the one hundred runs, confirmed by the low level of the related variance. Additional analyses on the sensibility to key parameters are provided in the Annexes.

4.1 Existence and effectiveness of the externalities.

The investigation is based on a comparison of the results obtained from running simulations of four scenarios (Alpha, Beta, Gamma and Iota), based on varying values of several key parameters: knowledge governance costs, the negative effects of knowledge appropriability on the price of innovated goods, and external opportunities, which influence the effects of localization in a commons on the accumulation of competence and the capability to absorb external knowledge.

In more detail, knowledge governance costs are computed for each firm according to the density of the commons to which they belong. Density exerts a non-linear effect so that knowledge governance costs vary according to the number of firms belonging to each commons following a U shape relation. In the first (Alpha scenario) this dynamic is fully at work, whereas in the other three (Beta, Gamma and Iota knowledge governance cost is to zero. The external opportunity parameter measures the effects of the productivity external to each agent, which adds to each agent's internal knowledge stock, at each production cycle. According to our model firms localized in a high productivity common accumulate more competence than firms localized in a low productivity one. This parameter takes three different values: i) in the Alpha and Theta scenarios it is set to 0.001 times the average productivity of the agents in the commons plus one; ii) in the Beta scenario, the experience accumulated in each production cycle is set to zero, that is, there is no cumulated experience; iii) in the Gamma and Iota scenarios, which mainly test the effectiveness of different setups for this parameter, the firms accumulate 0.001 of experience for whatever productivity levels achieved in the commons. Table 1 presents the experimental set ups, where N is the total number of enterprises in the economy, n is the number of enterprises belonging to a single common, and cp is the average productivity of all the firms belonging to a commons.

Table 1: Alpha versus others - set up of the different scenario.

Scenario	Number of commons	Common cost	Internal learnng	External learning
Alpha	10	(0.01* N)/n+0.01*n	0.001 *(1 + cp)	Yes
Beta	10	zero	zero	Yes
Gamma	10	zero	0.001	Yes
Iota	10	zero	0.001	No
Theta	4	(0.01* N)/n+0.01*n	0.001 *(1 + cp)	Yes

As Table 1 shows, knowledge governance costs are set to zero in the Beta, Gamma and Iota scenarios, and in the Alpha scenario, are allowed to vary according to the magnitude of each commons' population by following a U shaped relation. In the Alpha scenario firms achieve a larger accumulation of competence that reflects both the average productivity of the commons in which they are localized, and its productivity peaks. However, in the Alpha scenario, firms are liable for knowledge governance costs that vary according to the density of the commons (see Table 2).

Table 2: Alpha versus others – population and knowledge governance costs.

Scenario	Population of the common							
	1	50	100	150	250	500	750	1,000
Alpha	10.01	0.70	1.10	1.57	2.54	5.02	7.51	10.01
Beta	zero	zero	zero	zero	zero	zero	zero	zero
Gamma	zero	zero	zero	zero	zero	zero	zero	zero
Iota	zero	zero	zero	zero	zero	zero	zero	zero
Theta	10.01	0.70	1.10	1.57	2.54	5.02	7.51	10.01

The main simulation result is based on a comparison of productivity growth across the three sets of parameters. We expect the Alpha scenario to exhibit the best performance. The interpretation of the results is straightforward: i) the Beta scenario tests the generic importance of knowledge in determining the dynamics of productivity and production; we expect the poorest results from the Beta scenario; ii) the Gamma scenario will negate our hypothesis if its results were close to those from the Alpha scenario; and iii) the Iota scenario underlines the dramatic importance of spillovers for the growth of knowledge and productivity. We observe that the three alternative scenarios do not overtake the performance of the Alpha scenario where knowledge externalities are fully at work. Table 3 shows the average results of one hundred simulations for each scenario: the evidence confirms that the results are not dependent upon random distribution due to the meaningless level of variance among the one hundred simulation trials even when they were based upon different random seeded distributions.

Table 3: Alpha versus others - macro system level productivity.

Scenario	Min productivity	Average Productivity	Max Productivity	Variance
Alpha	17.569	22.717	24.083	0.6003078
Beta	0.249	0.250	0.250	0.0000000
Gamma	16.195	16.805	17.209	0.0430414
Iota	1.395	1.416	1.449	0.0001804
Theta	20.002	22.829	24.175	0.7206033

After 2,000 production cycles, in the Alpha scenario, the system, as a collection of commons, reaches an average productivity of 22.717; in the same number of simulation steps, in the Gamma, Iota and Beta scenarios, the system reaches, respectively 16.805, 1.416 and 0.25. The Theta scenario, based upon the same parameter configuration of the alpha ones, differs only for the number of commons:

their results are very closed to them of the alpha ones. The number of commons seems to have very little influence on the results at the macro system level.
A batch of one hundred simulations has run respectively for alpha and gamma scenario, with an higher cost of labour – there wages were set to 10 instead of 1 – in order to test that the alpha scenario drove to higher productivity level than the gamma one, independently from the labour cost. The comparison among the two scenarios confirms the importance of knowledge externalities, even the distance between the final productivity achieved by the two scenario was less: whereas the alpha scenario reached a productivity of 20.585 (average of the values reached in the one hundred simulations) the gamma scenario stop its performance at the level of 16.626.
Table 4 reports the minimum and maximum populations achieved during the first 2000 production cycles across the ten commons economy, as well as the dynamic due to moving across commons by means of the minimum and maximum turnover – i. e. the sum of enterprises that had entered the commons and had gone away from it. Again, the interpretation is straightforward: the structure of the system is endogenous. There is a clear technological and structural change loop. We see that the pace of productivity at system level is affected by the distribution of firms across commons. At the same time, the structure of the system is affected by the different dynamics of productivity. The loop encompasses historic time and leads to strong non-ergodic path dependence. The Alpha scenario, with strong positive knowledge externalities fully at work, shows lower levels of concentration of firms across commons. Concentrations are greater in the scenarios where the effects of externalities on competence are smaller, and naturally where the number of commons is limited like in the scenario Theta. Commons-to-commons flows are dramatically higher for the Iota scenario where firms cannot engage in external learning so react to out-of-equilibrium conditions by moving continuously from commons to commons.

Table 4: Alpha versus others – commons min and max population and turnover

Scenario	Min Size	Max Size	Min Turnover	Max Turnover
Alpha	24	265	3,768	9,695
Beta	0	529	145	1,190
Gamma	27	277	7,054	13,319
Iota	66	141	29,095	31,555
Theta	57	537	11,501	24,440

Sensitivity to the key parameters does not raise concern. A few simulations have been devoted to test the sensitivity to four parameters that were expected to have strong influence, or to might have, on the results of the simulations. The table 5 briefly resumes the Pearson's index values computed through one hundred simulations run under randomly set up values for the parameters: i) tolerance – used the equilibrium condition: an agent is considered as "in equilibrium" if its results are different from the average ones more than tolerance, either in negative or positive terms -, IPR duration – the number of production cycles a technology enhancement is hidden to other firms due to IPR protection -, iii) Potential per step – the knowledge an enterprise accumulates each step due to learning by doing, iv) commonsCost – the base value for commons costs, both knowledge management and exploration, computation. The Pearson ratios have been computed between the random value of the parameter and the productivity level achieved after 2,000 production cycles at the macro system level, under constant values for each other parameter and same distribution of the random events.

Table 5 – Sensitivity to key parameters values

Scenario	Tolerance	IPR Duration	Internal Learning	Common Cost
Alpha	0.060	-0.873	0.510	-0.962

As the table 5 shows the longer the IPR protection lasts the littler the productivity level the system achieved after 2,000 production cycles, the same effect is shown for the common costs, even stronger. A correlation has been found with the learning capability that is a trivial but highly plausible remark. The tolerance level demonstrated to have a very weak correlation; levels tested were from 0 to 0.001 – the level usual employed for the simulations – in order to demonstrate that even littler set up for this parameter would have add very few to the meaningfulness of the simulations.

The essential remark of table 5 is in that the strong correlation between achieved productivity and, respectively, i) IPR duration and ii) Commons cost – i.e. knowledge governance cost – demonstrates that under poor or null knowledge externalities the behaviour of enterprises is doomed to be simply adaptive. This remark constitutes the ultimate answer of the research question this paper is based upon: the results confirm the claim for the dramatic effects of the endogenous dynamics of knowledge externalities. The analysis of the productivity growth in the different scenario, highlights the dramatic gaps among the four scenarios for average firm output, which is highest in the Alpha scenario.

The Alpha scenario exhibits faster rates of productivity growth, and a typical step-wise pattern of growth with periods of fast growth followed by phases of slow growth. Figure 1 shows that the availability of net positive knowledge externalities within each commons cum the mobility of firms across commons, stylized in the Alpha scenario, are able to push the whole economy to far higher productivity values.

Figure 1: Alpha versus others – macro system level productivity.

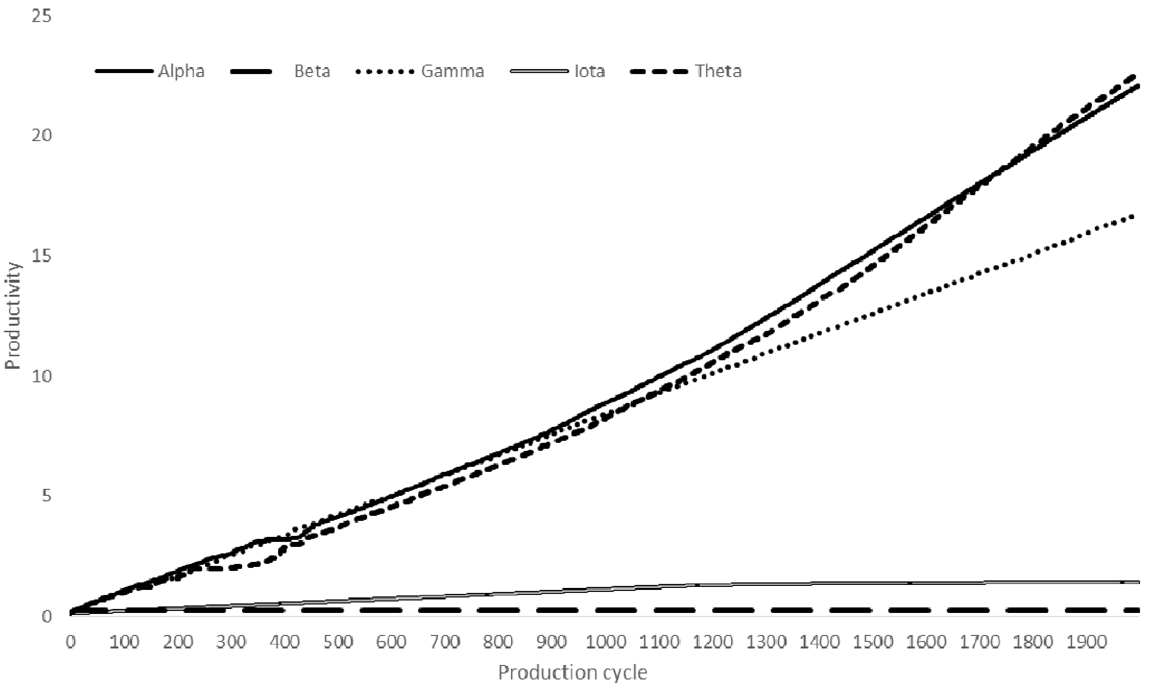
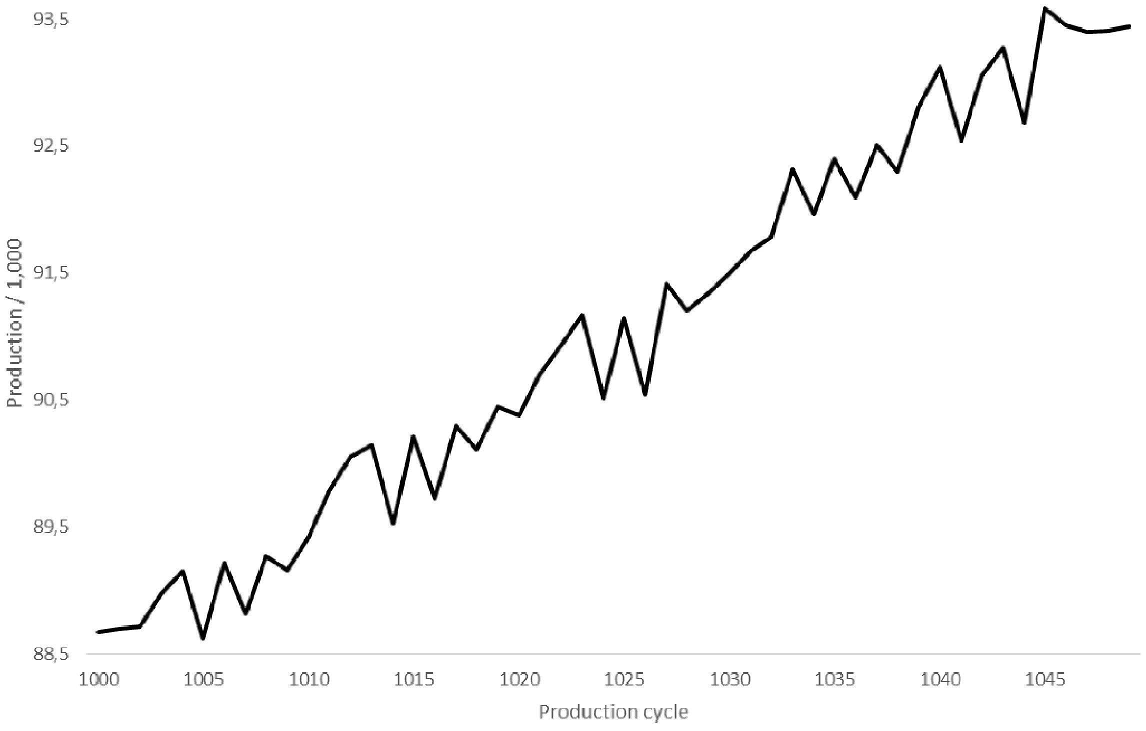


Figure 1 shows that the availability of net positive knowledge externalities within each commons cum the mobility of firms across commons, stylized in the Alpha scenario, are able to push the whole economy to far higher productivity values. Consistent with this, output at the macro system level shows larger growth in the Alpha scenario compared to the others. Figure 2 highlights the typical step-wise pattern of growth when knowledge externalities are fully at work – i.e. Alpha scenario - with periods of fast growth followed by phases of slow growth.

Figure 2: Output level during a period of fifty production cycles.

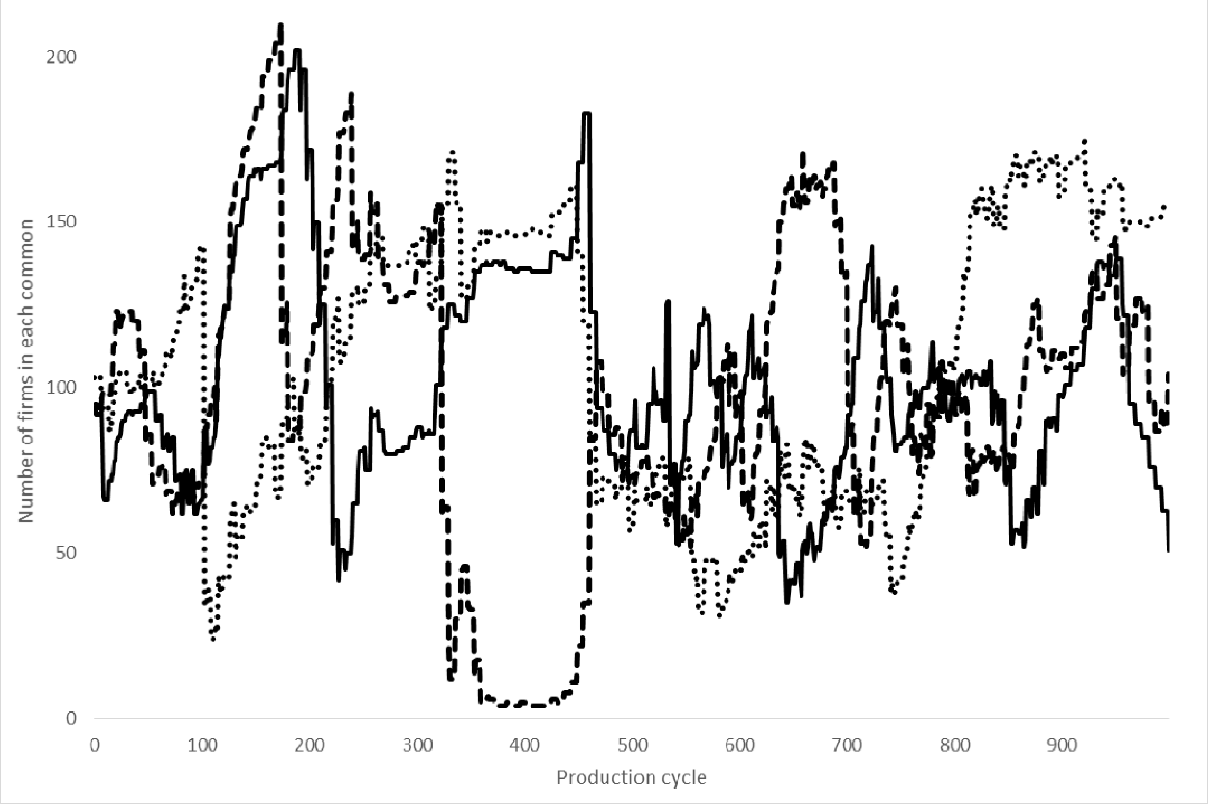


4.2 The dynamics of the commons in the Alpha scenario.

The Alpha scenario represents our benchmark, validated by the results of the previous simulations. It is interesting to explore the dynamics of structural change engendered by the model of creative response cum knowledge externalities at the commons level.

At commons level, the results of the simulation show that, although selection of a new commons is a blind activity for agents, their mobility strongly affects the structure of the system and the size of each commons. Figure 3 provides a general representation of the phenomenon at commons level by showing the number of firms in the first three commons during one thousand production cycles. It shows clearly that each commons undergoes a typical Schumpeterian wave, with phases of growth and subsequent decline along the process. The long-term pattern of growth is punctuated by waves where, after rapid take-off, the commons enters a contraction phase, due to the rising knowledge governance costs for excessive crowding. As one commons contracts, others increase in size - of output and number of firms.

Figure 3: Alpha - Waves of population patterns across commons.



Over the long term, the oscillations level out and the size of commons become increasingly homogeneous with a clear decline in concentration. Variety among commons seems to exert a strong and positive effect on the overall increase in productivity at system level. This evidence warrants further analysis, but could be considered to hint at the powerful effects of replicator dynamics according to which the rate of growth of a system is positively influenced by its variety (Metcalfe, 2002). The Schumpeterian waves at commons level affect overall aggregate patterns of productivity growth at system level, which show a typical step-wise pattern (see Figure 2). The evidence from these simulations hints at an innovation process conceived as a Schumpeterian creative reaction enabled by knowledge externalities, engendering structural change and 'disorder' at commons level, with marked Schumpeterian waves of output growth and firm populations, which positively affect the system level dynamics where both output and productivity show continuous step-wise growth. Creative destruction occurs at the firm and commons levels, but benefits the system at large. The locus of innovation shifts along time from one commons to another, in a punctuated sequence that closely parallels the long-term historic trends identified by Mokyr (1990).

5. Conclusions and policy implications

The understanding of the pervasive role of the Arrovian properties of knowledge as an economic good: non-appropriability, non-exhaustibility, and cumulability and complementarity stemming from its indivisibility, makes it possible to grasp the recombinant character of its generation process. This process involves external knowledge as an indispensable input in the generation of new knowledge and the eventual introduction of innovations. The creative reaction of firms caught in out-of-equilibrium conditions and the necessary generation of knowledge is enabled by the pecuniary knowledge externalities stemming from the quality and structure of the networks of synchronic and diachronic complementarities among firms linked by formal and informal ties. However, pecuniary knowledge externalities are not always available. The success of the creative reactions of firms caught in out-of-equilibrium conditions, to generate new technological knowledge and introduce productivity enhancing innovations, depends on the availability of pecuniary knowledge externalities. In these intrinsically localized circumstances, innovation is a highly specific and idiosyncratic emerging property that takes place only when the complexity of the local system is properly organized and adequate levels of knowledge connectivity are reached and maintained. The success of such creative reactions, in turn, changes the organization of the system and its knowledge connectivity and may reinforce the availability of pecuniary knowledge externalities, feeding a self-sustained process of growth and change, as well as endangering it. The process is far from deterministic: excess density with the consequent decline of knowledge connectivity, in fact, is a possible outcome of the generation of additional knowledge and the changes in the structure of the system that stem from the introduction of innovations.

Knowledge externalities are endogenous: there is a causal loop linking the amount of knowledge that each firm can generate with the cost of available external knowledge, including knowledge governance costs, which, in turn, depend upon the – changing - structure of interactions and transactions, and density of co-localized firms. The larger the pecuniary knowledge externalities, the stronger are the incentives for firms to try to enter knowledge-rich commons. Their entry affects the knowledge connectivity of the system and hence its knowledge governance costs as well as the supply of technological spillovers, and changes the level of the available pecuniary knowledge externalities.

The stock of external knowledge available at any point in time, and in regional and technological space, is not determined by exogenous factors, but is strongly influenced by the conditions of knowledge governance costs within the knowledge commons, as well as by the amount of creative reactions that have been taking place at each point in time.

The use of an ABM allows to articulate the relations between the basic ingredients of the dynamic processes, and to elaborate a coherent analytical framework that helps to explain, and mimics the endogenous long term dynamics of technological and structural change that are at the heart of economic growth. Thus, the ABM can be considered a type of artificial cliometrics, providing the opportunity to test a set of hypotheses about the role of endogenous knowledge externalities. The results of the ABM confirm that endogenous knowledge externalities have powerful effects on the equilibrium conditions of the system dynamics at the micro-, meso- and macro-levels.

At the micro-level we show that the reaction of firms caught in out-of-equilibrium conditions yields successful effects, with the introduction of productivity enhancing innovations, when pecuniary knowledge externalities provide by high levels of knowledge connectivity are available. Innovation is the result of matching individual and intentional learning efforts in reactive agents with the characteristics of the system in which the firm is embedded. Innovation is an emerging property of the system, in which individual action is as indispensable as the availability of positive pecuniary knowledge externalities. Endogenous knowledge externalities generate endogenous growth characterized intrinsically by an out-of-equilibrium state. The introduction of innovation affects the transient equilibrium of product and factor markets, exposes each firm to changes in its relative profitability, and induces new innovation efforts. Equilibrium occurs only if and when innovation is impossible because of lack of pecuniary knowledge externalities. Innovation and equilibrium are antithetical.

At the meso-level, the out-of-equilibrium dynamics of endogenous knowledge externalities affect the structural characteristics of the commons and the aggregate system. Endogenous centrifugal and centripetal forces continually re-shape each commons and the structure of the system, and produce ever-changing heterogeneity characterized by the creation and decline of knowledge commons. The process exhibits the typical traits of a third order emergence where micro processes lead to aggregate changes that in turn –may- affect the likelihood of the microdynamics (Martin and Sunley, 2012). To try and access pecuniary knowledge externalities, firms can move across commons. This mobility may have the twin effect to: a) increase their chances to innovate and b) change the structural landscape and the consequent levels of knowledge connectivity of the each commons and, hence, of the system, viewed as a collection of commons. Within commons, the mobility across commons affects local knowledge governance costs and changes the levels of pecuniary knowledge externalities and, thus, the likelihood that co-localized firms can generate new technological knowledge and introduce technological innovations which will increase their productivity. A knowledge commons, endowed with firms that enjoy high levels of productivity, may attract many learning firms willing to improve their productivity. Their entry, however, may affect the local levels of knowledge governance costs and reduce the levels of net positive pecuniary knowledge externalities, reducing the overall attractiveness of the location and the aggregate dynamics of the system. Local systems may experience a transition from high levels of organized complexity able to generate high levels of net positive knowledge externalities, to low levels of organized complexity where congestion and governance costs make the access to knowledge spillovers more expensive.

At the single commons level, the out-of-equilibrium process leads to non-linear patterns of economic growth characterized by significant oscillations in the firm population levels, and rates of output, profitability and productivity growth, that take the form typical of long waves in Schumpeterian analyses of business cycles.

At the system level, the dynamics of productivity growth exhibits a typical step-wise pattern with long periods of time characterized by smooth rates of increase, and sudden, sharp jumps. When the distribution of firms within the knowledge commons is particularly effective, and the local system is able to promote high levels of knowledge externalities, the rate of generation of new knowledge and the rate of productivity enhancing innovation increase. At the aggregate level, the system experiences fast rates of output and productivity growth. In the opposite case, the distribution of the firms across knowledge commons reduces the opportunities to benefit from net positive knowledge externalities. Crowded knowledge commons command high levels of knowledge governance costs, and peripheral knowledge commons with low levels of productivity involve few opportunities for knowledge dissemination, and the system experiences low rates of innovation introduction and productivity growth.

The endogenous dynamics of knowledge externalities engenders multiple equilibria as well as micro-macro feedbacks such that the dynamics of the system becomes very sensitive to small and unintended shocks. In the case of a single attractor, prices perform as vectors of reliable signals about markets conditions, and competition restores the equilibrium conditions. In the opposite case, in a dynamic context based on out-of-equilibrium conditions, the consequences of individual action on the structural characteristics of the system are difficult to foresee. In the local context and over a short time span, only procedural rationality will apply. There is no countervailing force that can identify a real attractor. Therefore, entrepreneurial action may have major consequences at the economic system level with either positive or negative effects. Access to external knowledge, and dissemination of knowledge generally, are far from being automatic. They are stochastic not deterministic processes, and may or may not occur depending on the characteristics of the system that are not given only once and are not exogenous, but rather are constantly changing through time as a consequence of agents’ actions.

The endogenous dynamics of pecuniary knowledge externalities is intrinsically path dependent. The existing structure of the system affects the dynamics, but at each point in time firms can change the amounts of resources invested in the generation of knowledge, new governance mechanisms can be introduced, and the mobility of firms across the knowledge and regional space changes the structure of the system and the levels of pecuniary knowledge externalities.

The policy implications of these results are important in highlighting the endogenous dynamics of knowledge externalities. Knowledge externalities do not fall like manna from heaven and are not given once and forever. There has been an extreme focus on knowledge generating policies to the detriment of policies for knowledge governance. Careful policy interventions to promote intentional changes to the system parameters in order to improve knowledge governance could have long-lasting and positive effects (Ostrom and Hess, 2006; Ostrom, 2010).

Knowledge dissemination should become the topic of dedicated policies aimed at favouring the access and use of external knowledge as an indispensable input to the successful recombinant generation of new technological knowledge. The design of specific applications of new IPR regimes that favour knowledge dissemination and yet enable appropriate levels of knowledge appropriability could be very effective for knowledge dissemination. Systematic introduction of measures that would reduce exclusive property rights based upon compulsory licencing with fair royalties would likely have strong positive effects on the rates of generation of new technological knowledge. We would stress here that, although the implementation of interventions affecting the basic architecture of IPR regimes might seem rather controversial, our argument becomes more realistic and palatable when considered as the introduction of non-exclusive IPR for patents stemming from public interventions, ranging from public procurement to research activities supported by public funding. Interventions that support the purchase of patents and, more generally, interactions between knowledge producers and knowledge users and all public subsidies, would help the dissemination of knowledge both within core regions and among regions (Reichman, 2000).

Support for mobility of skilled personnel can be a very effective tool for knowledge dissemination. In core regions it would help to reduce knowledge absorption costs, and across regions support for mobility of skilled personnel, academics and inventors from core regions can make (re)location in a semi-core region more attractive. This type of support would favour interregional knowledge dissemination from core to non-core regions.

The strengthening of effective interactions between firms and the academic system both within and across commons is likely to reduce substantially the costs of external knowledge. The entry of the public research infrastructure in the market for knowledge outsourcing can help the effective absorption of knowledge generated by the public research infrastructure increasing the actual amount of net positive knowledge externalities. The public research infrastructure can take advantage of the signals provided by firms so as to better direct the internal inter-disciplinary allocation of resources. Firms can access the research capabilities of large and effective public R&D labs to perform R&D activities taking advantage of substantial increasing returns and low unit fixed costs. The implementation of effective systems of interaction can improve the matching between the public research infrastructure and the business community so as to increase the amount of net positive knowledge externalities available in the system favouring its growth dynamics.

Support for the creation of academic networks between strong academic institutes in core-regions, linked by strong institutional ties, and peripheral universities located in non-core regions would help the dissemination of knowledge across regions. Within regions, dissemination of academic knowledge would be increased by reducing the exclusivity in academic employment contracts to allow individual academics to participate knowledge consulting activities. Especially for small firms, using academics as consultants would allow them to build contractual relations in knowledge typical of large corporations. All interventions that increase the access to external knowledge and reduce knowledge interaction costs are likely to exert positive effects on the dynamics of economic systems.

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APPENDIX A – The pseudo code of the model

(Parameter of the simulation are written in italic bold)

```
Repeat-until the end of the simulation
Each firm      If (status is “in business”): send order to the market to buy work.
The market     Compute sell prices for work by setting the constant value 1
Each firm      If (status is “in business”):
                   Compute wages to pay as (labour for production + labour for research) * price of work.
                   Compute output as: work for production * productivity.
                   Increase amount of internal competences by: common’s productivitiy * potentialPerStep.
                   Offer the whole output into the market.
               End-If
Each worker    If (wealth > 0): spend whole wealth to buy product. Else spend nothing. End-If
The market     Compute sell prices for product as: demand / supply.
Each firm      If (status is “in business”):
                   Compute income as: production * sell price.
                   Compute profit as: income – wages.
                   Pay wages by sending the workers the message cashWages
                   Distribute dividends by sending the worker the message cashDividens.
                   If (losses are greater than maxLosses) set status to “out of business”.
               End-If
Each common    Compute the average amount of production, profits and productivity.
               If (commonCost is set greater than zero): compute knowledge governance costs in working units for research as: total number of firms * commonCost / number of firms into the common + number of firms into the common times commonCost.
               Report fluxes of agents in the latest cycle.
The model      Compute aggregate statistics at the Macro System Level.
Each firm      If (status is “in business”):
                   If (profit > 0 + tolerance)
                       increase demand by factorUp
                       assign the whole profit to the investment budget
                   End-If
                   If (profit < 0 - tolerance)
                       reduce demand by factorUp
                       assign the saved factor to the research budget
                   End-If
                   Compute investment budget as profit / wages, if pr
                   Compute lowerThreshold as common’s average profit * (1-tolerance).
                   Compute upperThreshold as common’s average profit * (1+tolerance).
                   If (profit is grater than upperThreshold):
                       increase number of successes
```

```
            set to zero the number of failures.
        End-If
        If (profit is less than lowerThreshold)
            increase number of failures
            set to zero the number of successes.
        End-If
        If (number of failures is greater than failuresThreshold) or
            (number of successes is greater than successesThreshold):
            try to exploit internal competence.
            If (trial fails) try local absorption of external knowledge into the common.
            If(local absorption fails) move randomly to another common
        End-If.
    End-If.
End-If.
Each commons    Compute the average amount of production, profits and productivity.
Each agent    If (status is "out of business"): Increase the counter of steps in "out of Business" status
    If (steps out of business are greater than revampTime):
        Set productivity to the average common's one
        Upgrade the IPR status of patents.
        Set status to "in business".
    End-If
Each commons    Compute the average amount of production, profits and productivity.
End-repeat-until
```

APPENDIX B –Parameters of the model and set up for the different simulations

In order to control the simulations and allow configuration of a wide set of different scenarios, a reach set of parameter has been provided to bias both the behaviour of agents (firms) and the structure of the economic at system macro level and common macro level too. In the simulations of this paper few of the available parameters vary, their number has been set to support further evolutions of the research, so many parameters have had the same value for all the simulations presented in the paper, related to those parameter no sensitivity analysis, neither specific simulations have been done due to the fact their values were always the same for each simulations or scenario. Models based on the Swarm protocol distinguish two different object devoted to control the simulation: the Observer that is charged to collect and report the results emerging during the simulation the Model that is charge to build all the objects to populate the model and schedule the activity of those ones. Both Observer and Model give the possibility to specify customized parameters,

The observer uses a first set of two parameters to determine the output shape and update:

- **displayFrequency** set up the interval, in model steps, between each refresh of the graphs. Because the presented simulation was devoted to study the dynamic of the system, this parameter has been set to 1 (i.e. graphs are redrawn at each simulation step) to fully report variations in the observed quantities.
- **zoomFactor** influence the shape of the graphs produced and updated during the simulation run. Its value is usually set to 2, all the simulations used that value.

The model has been provided a wider set of parameters to make the configuration of different scenario very easy. In details:

- **randomSeed:** (any natural number in the interval]0,∞[) it is used to initialize the random seed generator. Useful both to vary the random distributions as well as to ensure the possibility to replicate an experiment with the same random number distribution.
- **Agents:** (any natural number in the interval]0,∞[) determines the number of firms that will be put in the simulated economy. The maximum number of firms allowed for a simulation depends on the memory and processing power of the computer used for the simulation.
- **Commons:** (any natural number in the interval]0,∞[) specifies how many commons will be generate and used into the simulation. The maximum number of firms allowed for a simulation depends on the memory and processing power of the computer used for the simulation.
- **InitialWealth:** (any real number in the interval]0,∞[) specifies the initial endowment of workers they're going to offer into the market to buy the first productive cycle's output.

- **StartingProductivity:** (any real number in the interval]0, ∞[) indicates the upper limit for the interval used to assign each agent an initial productivity, by randomly tossing, for each, a different real number into the interval]0,startingProductivity[.
- **StartingFactor:** (any real number in the interval]1,∞[) specifies the quantity of work units the enterprises will demand on the market and employ for production in the first simulated production cycle.
- **SuccessesThreshold:** (any natural number in the interval]0,∞[) specifies how much consecutive successes have to be piled before starting a trial for innovation. A success is achieved every time the own profit of the agent is greater than the average common's one + a tolerance percentage.
- **FailuresThreshold:** (any natural number in the interval]0,∞[) ,∞[) specifies how much consecutive failures have to be piled before starting a trial for innovation. A failure is suffered every time the own profit of the agent is less than the average common's one - a tolerance percentage.
- **IprDuration:** (any natural number in the interval]0,∞[) specifies the number of production cycle the patent rights protect each innovation, during this time the innovation is hidden to the other agents.
- **RevampTime:** (any natural number in the interval]0,∞[) specifies the number of production cycle after an agent is gone out of business for having another one keep its place. The name of the parameter is due to the fact that a new agents is only the revamp of the old one, with productivity equal to the average common's one.
- **FactorUp:** (any real number in the interval]1,∞[) is the number used to multiply the previous demand for factor to determine the actual one, by the firms that achieved a profit. Because the base assumption is that profitable firms will expand the production this parameter have to be set at a value greater than one, but close to one; for instance setting factorUp to two would means that enterprises that had a profit will double their demand for factor (work) for the next production cycle. **FactorDown:** (any real number in the interval]0,1[) is the number used to multiply the previous demand for factor by the enterprises that just suffered a loss; this parameter has to be set close to one too, even less than one.
- **PotentialPerStep:** (any real number in the interval]0,1[) represents the fraction of the common's productivity that each agent accumulate in each production cycle due to experience. This parameter has to be set accordingly with the following "Productivity upgrade", that measure the amount of accumulated experience needed to enhance technology of one unit.
- **ProductivityUpgrade:** (any real number in the interval]0,∞[) represents the minimum quantity of accumulated potential that could be transformed in a unit of technological enhancement. Note that transformation can be performed for this amount of cumulated experience at a time only, and gives one unit of technological enhancement.

- **Tolerance:** (any real number in the interval]0,1[), defines the symmetric interval around the average common's profit used by each firm to decide if take actions to improve its technology. Unless the result (either profit or loss) of an enterprise in a certain production cycle was less than (average common's results * (1-tolerance)) or was grater than (average common's results * (1+tolerance)) no improvement on the technological level are tried.
- **MaxLoss:** (any real number in the interval]0,∞[), it measures the maximum loss an enterprises can cumulate before going out of business. The meaning is in that if cumulated results of an enterprises reach a negative amount less than (-1)maxLoss it goes immediately out of business and will be replaced, into the same common, by another one after revampTime production cycles.
- **TransformationCost:** (any real number in the interval [0,∞[), it measures the amount of work unit an enterprise has to demand on the market to perform a transformation of accumulated experience in technological enhancement.
- **MovingCost:** (any real number in the interval [0,∞[), it measures the amount of work unit an enterprise has to demand on the market to move from the actual common to another one.
- **SpilloverMinProb:** (any real number in the interval [0,1]), is the success base probability assigned to a generic spillover action, the effective probability of success for a spillover action is computed as: (1-spilloverMinProb)(1-delta)+spilloverMinProb), where delta measures the distance between productivity after and before the spillover action.
- **CommonsCost:** (any real number in the interval [0,∞[), is the amount used to compute the quantity of work each firms has to buy, each production cycle, to get information and manage relation into the commons it belongs to. The costs are the same for each agent into the commons.

In order to manage the simulations, some control parameters have been used:

- **Scenario:** (an integer number in the interval [1,5]) specifies the scenario to be executed among: Alpha, Beta, Gamma, Iota and Theta; by choosing one of them the program automatically sets up the core parameters to configure the chosen scenario.
- **Spillover:** (an integer number in the interval [0,1]) it is a simple switch that allows or stops the possibility for firms to spill knowledge from other ones into the common.
- **Reinforce:** (an integer number in the interval [0,1]) used to turn on or off the effect of the common productivity on the knowledge each enterprise grows up by executing each production cycle.

Finally the old parameters: i) **focused**, ii) **explorationRate** and iii) **spilloverCostRate** are no more used, even still present in the input form.

Appendix C – Parameters’ values for the simulations

The simulations used for the research were based upon different settings of few parameters, as described in chapter four, obtained by biasing a base configuration named alpha scenario. The full parameters setting for each scenario are reported in the following table F1.

Table F1 – Parameters’ setting for each scenario

	Alpha	Beta	Gamma	Iota	Theta
Agents	1000	1000	1000	1000	1000
Commons	10	10	10	10	4
Scenario	1	2	3	4	5
Spillover	1	1	1	0	1
Reinforce	1	0	0	0	1
SuccessThreshold	5	5	5	5	5
FailureThreshold	5	5	5	5	5
IprDuration	5	5	5	5	5
revampTime	10	10	10	10	10
startingFactor	100	100	100	100	100
factorUp	0.01	0.01	0.01	0.01	0.01
factorDown	0.01	0.01	0.01	0.01	0.01
potentialPerStep	0.001	0.001	0.001	0.001	0.001
productivityUpgrade	0.1	0.1	0.1	0.1	0.1
tolerance	0.001	0.001	0.001	0.001	0.001
maxLoss	50,000	50,000	50,000	50,000	50,000
explorationRate	1	1	1	1	1
transformationCost	1	1	1	1	1
movingCost	1	1	1	1	1
spilloverMinProb	0.8	0.8	0.8	0.8	0.8
commonCost	0.01	0.01	0.01	0.01	0.01
initialWealth	10,000,000	10,000,000	10,000,000	10,000,000	10,000,000
startingProductivity	0.25	0.25	0.25	0.25	0.25

Appendix D – Robustness and sensitivity

D1. Introduction – pseudo random generators

Several processes in the model are based on random events: i) technological improvement may fail according to a probability distribution set parametrically, ii) other firms are picked up randomly among the neighbours to observe and eventually imitate, iii) enterprises that move decide randomly the new common to enter in, etc. The generation of pseudo random numbers has to be managed with specific care.

In order to guarantee the full independence of each agent, as well as of each environmental component, like commons, market and so on, each object has been provided an own random generator (Ferraris 2006a and b); the control of the random distributions is based on a simple procedure:

a) The modelSwarm object (the main component that is charged to build and activate all the other ones, either agents or environmental institutions, has been provided with an own random generator (named the main generator), whose seed can be fixed by the researcher, simply supplying a value for the parameter "randomSeed". If the value zero is specified the model tosses a random seed using the standard generator the simulation tool (for this model Swarm) provided.
   Note that this generator is not used to toss random values for parameters expected to vary. In this way it is possible to fix all the random events, even with parameters that are randomly set up.
b) Each component of the model that uses random numbers is given an own random generator, made by the modelSwarm just before building the component, fed with a seed tossed by using the main generator.

Such a architecture allows both: i) independence of each component even for humongous numbers of requests for random values, ii) full control of the random generation. In this way the researcher is allowed to:

a) exploit the possibility to replicate a simulation with the same sequence of random numbers,
b) change randomly the sequence,
c) avoid interferences among agents and environmental institutions (or generally speaking components) even with heavy usage of random numbers.

The exploitation of the previous described architecture allowed both robustness and sensitivity tests, whose results are briefly described in the next paragraphs.

D2. Robustness

To ensure that the results obtained from the simulations were independent from the random distributions, a simple robustness test has been performed:

a) The model has been run for one hundred times with fixed parameters values but randomly changing, each time, the seed of the main generator (recalling the architecture described in F1, this means a different random seed, each time, for the more than one thousand objects involved in each simulations).

b) For each simulation the average values of the productivity are measured at the 2000th production cycles (because each simulations employed one thousand agents, it means 2 million production cycles for each measure, that was based on several millions random tossed values for different decisions).
c) After one hundred values of productivity obtained running the simulations, the mean, variance of the values, a succession has been computed to evaluate the independence of the results from the random seeds distribution.

Table E1 resumes the results: the variance of both productivity and commons dimension is quite low and seems to confirm the simulations results are not determined by employed random distributions, neither Pearson’s r value shows correlation between results and random seeds.

Table E1 – Analysis of the results obtained by one hundred simulations with same parameters’ values but different random seed, based upon the alpha scenario.

Measure	Value
Min Productivity	17.569
Max Productivity	24.083
Average Productivity	22.717
Productivity variance	0.600