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KNOWLEDGE CLUSTERS AND MULTIDIMENSIONAL PROXIMITY: AN AGENT-BASED SIMULATION

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Knowledge Clusters and Multidimensional Proximity

An Agent-Based Simulation

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We will use an Agent-Based Model in order to study how innovation can emerge from the interaction between firms. In particular, we are interested in studying how the clusters that emerges from these interactions influence the ability of bounded rational firms in reacting creatively to out-of-equilibrium conditions. Moreover, we will introduce two type of firms – traditional and innovative – and we will observe if and how this difference influences the outcomes.

Keywords: *Spillovers*; *Agent-Based Model*; *Industrial clusters.*

1. Introduction

Technological progress represents one of the main themes of present economic debate and is the object of one of the most important line of economic research of the last fifteen years.

It is widely supported that, in the post-Fordist world, innovation has become the key resource for the firms of the most advanced and developed countries. However, for its own characteristics, innovation involves high costs and risks that cannot be taken on by each firm alone: they must share them being more open to external collaborations. In such an economic system, relations come back to be a relevant economic factor and therefore the co-localization is again a profitable strategy [Trigilia, 2007]. However, in the era of the so called "knowledge-based economy" the dematerialization of the production let the learning and sharing of tacit knowledge activities not strongly subject to the frictions of geographical distance: also other proximity dimensions seem relevant, in the first place the *relational* or *social* one. More in general, the different proximities reduce uncertainty and coordination problems, and, doing so, they make easier interactive learning and innovation [Boschma, 2005].

Within these places there exists external economies which both help in decreasing innovation costs and increase the probability of success of each firm in being «creative». Innovation becomes more fragmented and dispersed, as if there were *knowledge packages* that can be decomposed and recomposed in order to obtain something new also in lack of a radical technological revolution.

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Nevertheless, the economic literature seems not been able to fully reproduce the stylized facts an economic system shows when dynamically considered. Specifically, even when innovation was considered as a factor of economic growth (e.g. in the Endogenous Growth theory), Economics lacks an explanation of the *generation* of innovation as an endogenous product of the forces of the economic system itself [Mokyr, 2002]. Firstly, as already underlined by Schumpeter, a relevant limit is the use of a General Equilibrium framework, since within such a theory the only possible growth is quantitative. In general equilibrium economics the preferences and the technologies of the representative agent, and hence his production and utility functions, are allowed to change only as the result of exogenous shocks. As soon as the idea that new products can endogenously appear in the economic system is introduced and heterogeneous agents are endowed with the ability to change their production and utility functions in response to economic events, the foundations of general equilibrium economics collapse [Antonelli, 2011b]. Moreover, innovation was mostly treated as the product of an intermediate stage of production, without thereby being able to give an account of his relevant extra-economic components. Besides, the complete disregard for the idea of *exaptation* [Gould and Vrba, 1982] – with an interpretation of evolutions uniquely as *adaptations* – do not account for those innovations obtained by remixing already known pieces of knowledge [Mokyr, 1990, 1998].

Conversely, as shown by Antonelli, in order to explain innovation as an endogenous product of the economy itself, it seems more effective to adopt a framework of analysis that takes into account the marshallian Partial Equilibrium approach and the schumpeterian dyinamic analysis [Antonelli, 2011b]. The economy described trough the so called Localized Technological Change framework [Antonelli, 2007] can easily be seen as a Complex Adaptive System (CAS) [Holland, 1987; Holland and Miller, 1991], where *innovation* is considered an *emerging property* of an economic system: knowledge interactions among heterogeneous agents are crucial for the recombinant generation of new technological knowledge, and are these very *knowledge packages* that will be transformed in novelties TFP-increasing. Therefore, a paradigm shift from Neoclassical equipments to Complexity theory instruments is quite natural [Dawid, 2006; Fontana, 2008; Pyka and Fagiolo, 2007].

2. Methodological framework

In particular, the Agent-Based simulation Models (ABM) are more capable to reproduce these characteristics of dispersed innovation systems based on direct knowledge interactions. The introduction of such instruments in Economics is not a novelty and many models exists by more then two decades. Nevertheless, their diffusion in the Economics of Innovation and Technological Change field has not been wide and is begun only more recently [Dawid, 2006]. Among many others, we can think about the models by Antonelli and Ferraris [2011, 2012]; Dosi, Fagiolo, et al. [2010]; Fagiolo and Dosi [2003]. They not only reproduce many stylized facts that economic systems show when dynamically considered – specifically in terms of business cycles and abrupt strong fluctuations – but also allow to compare the effect of different institutional arrangements on the evolution of the system.

One main advantage of using this type of tools arises from the fact that here is the environment to be very complex, so people do not need to be very complicated, as well as the Simon's ant walking on the sand [H. A. Simon, 1969]. They are still problem-solving oriented individuals. However, within the end-means schemes they use, the ends may be gradually modified during the process and adapted to changing situations. This activity is a "trial and error" heuristic perfectly compatible with an agents' procedural rationality [Lanzara, 1988; March and H. Simon, 1958; H. A. Simon, 1969] and the evolution of such a system is mainly guided by «exaptations».

Once the *problem space–solution space* linkage is break down we cannot use a Neoclassical framework anymore, since we need to have well defined ends in order to apply the well-known definition of Economics by Robbins. Conversely, after this break economies with different institutions or technologies do not need to be reproduced by different models, but they can represent just different points in time and space of the same economy where the internal dynamics of the model are those that change the system itself. The use of tools that do not need of a strong aprioristic definition of the goals that agents want to achieve is needful in order to explain the generation of novelties in the system. Indeed, it is impossible to foresee the content of innovations to be made in the future (otherwise it would not be a novelty) and, therefore, to anticipate all possible directions of future technological development. This means that the system can be affected by *out-of-equilibrium conditions* for two basic reasons. Firstly, any economic environment influenced by innovations is characterized by "strong substantive uncertainty" [Dosi and Egidi, 1991; Dosi, Marengo, et al., 2003]. Moreover, as we have said, bounded rationality implies limited attention [H. A. Simon, 1969].

3. General purpose and analytical framework

Therefore, in order to reproduce such a complex system we use an Agent-Based Model where the innovation process is a circular dynamic in which knowledge is seen both as a (needful) input and as the output of the production process [Antonelli, 2011a]. As in Antonelli and Ferraris [2011], the general purpose of the model we are developing is to study how innovation can emerge from the interaction among firms.

In our model, firms have «creative reactions» when they are in *out-of-equilibrium* conditions [Antonelli, 2007; Antonelli and Scellato, 2008]. If their profits are much less or much more than the average of those of their neighbours, the firms react by trying to modify they "production function". To do so they must try to produce new pieces of knowledge, and this is possible only "if, when and where the generation of new technological knowledge and the eventual introduction of new technologies are supported by the actual availability of external knowledge to be used as an essential and indispensable production factor" [Antonelli, 2011a, p. 8]. In particular, when the firm tries to create innovation, it happens with a certain probability: the more the available knowledge around them and the more their expenditures in R&D activities, the higher the probability they succeed.

The amount of knowledge a firm can use to increase its probability of success is composed not only by its own knowledge. Through interaction with other firms each agent can collect a greater amount of knowledge. Not all the pieces of knowledge are available for a firm: they must belong to a firm that is in the neighbourhood of the exploiter, must be in the *public domain* and must be similar to the already owned knowledge.

As a mater of fact, the generation of new technological knowledge by each firm consists in the recombination of existing modules of knowledge. At each point in time the system is endowed with a given amount of technological knowledge characterized by high levels of heterogeneity and diversity both with respect to its epistemic content and location. Moreover it is possessed by the myriad of agents that generated it and are generating it. Therefore, since much technological knowledge is external to each agent they can increase the available amount of inputs through knowledge interactions with other economic subjects: firms can recombine internal and external pieces of knowledge in order to obtain an innovation [Arthur, 2009].

As underlined by Antonelli [2011a], existing technological knowledge cannot be used freely. Indeed, knowledge is dispersed in different contexts of application, it is codified in a variety of codes and owned by many different agents. This implies that existing external technological knowledge can be used in the recombinant generation of new technological knowledge only after dedicated resources have been invested.[1] Given that active search, screening, identification and interpretation of existing knowledge are necessary in order to use it again as an intermediary input into the production of new knowledge, the localized context of action emerges as a fundamental aspect of the innovation process since each firm can exploit only these external pieces of knowledge that are not too far from their already owned knowledge.

4. The Model

In our model firms lie on a 3D space (see fig. 1). Firstly, a dimension represents the technological or characteristic proximity of firms' production that is divided into some industrial sectors (represented by the different colours of the firms). The other two represent the two components of a geographical and relational map (represented by the plane).[2]

The possibility of this type of interaction is constrained, not only by the presence of IPRs on some pieces of knowledge,[3] but also by the strength of the network between the firms that own this knowledge. In order to increase their access to a greater amount of knowledge, the firms can change their relational location so to strengthen their network. The firms "sniff" each other and if they "smell" that there are some interesting amount of useful knowledge in a certain place of the plane they try to move there, with a mechanism similar to a "gravitational model" (see appendix A.2.4).

Innovative and traditional firms In our model each firm is different from the others because of its localization in the spaces, its budget, its R&D inclination, its reaction threshold and so on. Nevertheless, there exist two type of firms with different characteristics. Some firms, called *innovative* and represented as "factories"

[1] See Cohen and Levinthal [1990] for the idea of «absorptive capacity» and Bellandi [2003] for the idea of «specific public good».

[2] Two firms lie close to each other on this plane both because of their geographical localisation and because of the strength of their interactions that increases the more they collaborate.

[3] We tested the model for different levels of IPRs strength, that is for how long a new piece of knowledge is available only for the firm that generated it.

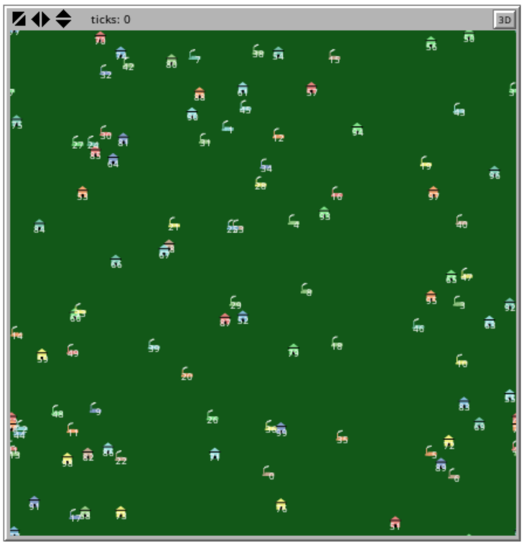


Figure 1: Initial setup example.

in the NetLogo implementation, have pieces of knowledge more scattered in the industrial sectors and, in order to innovate, they can access to a larger set of external knowledge.[4] In order to decide where to move this type of firm consider the pieces of knowledge of the other firms not only within their own industrial sector but also in the adjacent ones. Lastly, *innovative firms* tend to spend a large part of their budget in “creative” activities and have a lower profits threshold that drives them to innovate. The opposite holds for those firms called *traditional firms* and represented as “houses”.

Demand side (consumption) Some points on the plane represent a consumer or a group of consumers. These consumers buy the products of the firms in a approximated way through which we try to simulate the idea that the firms prefer to stay in a place with a high demand level and that the consumers prefer the variety but have also a decreasing marginal demand.[5] This process should be seen as complementary to the other movement the firms can do. The firms follow two different and conflicting heuristic exploration processes of their surrounding space:[6] one that leads them to get closer to other firms, forming clusters that help them to lower their «creative reaction» costs; the other that pushes them to escape price competition and expanding their demand size, so increasing their profits.[7]

[4] In some way, with the firms we have called *innovative*, we want to represent those firms that are often called *high-tech* firms [as in Trigilia, 2002].

[5] Even if in this way nothing is said with respect to the fact that also the type of product that a firm sell matters.

[6] Our firms are not *profit maximisers*, both because their reaction function is not continuous and because they have not perfect information.

[7] The idea is similar, with all the appropriate distinctions, to that used by Mai and Peng [1999] or by Piga and Poyago-Theotoky [2005] in their models *à la* Hotelling with spillovers between the two firms.

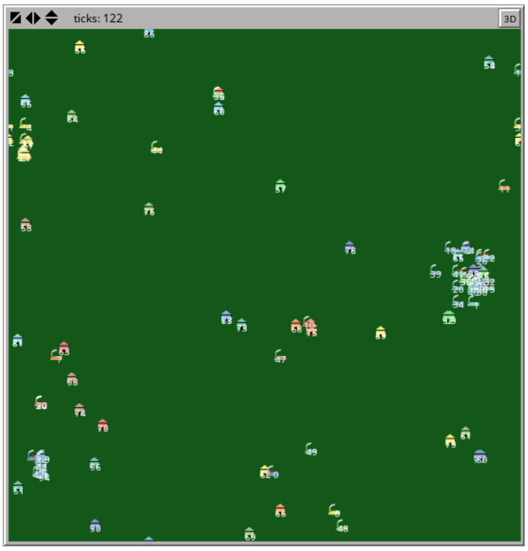


Figure 2: Emergent clusters example.

4.1. Specific purposes and research questions

So, more specifically, one of the specific purpose of the model is to analyse the differences in behaviour of the two types of firms. Therefore trough this model we try to analyse if the *innovative firms* tend to stay in *clusters* more than the *traditional* ones and if the number of firms in a *clusters* is affected by the proportion of *innovative firms* that are present in the simulated world.

Other research questions concern the actual ability of the *innovative firms* to produce more new pieces of knowledge than the *traditional firms* and the condition under which this is actually true. In particular we want to analyse whether the proportion of *innovative firms* in the simulated world affect the total number of new pieces of knowledge produced or not.

Lastly, we will look at if being in a cluster affect the ability of a firm to produce new pieces of knowledge and, provided that it is true, if there is a correlation between the ability to produce new knowledge and the technological sector which the firms that are in the cluster belong to.

5. Preliminary results

Generally speaking, simulations shown that *innovative firms* need to be in cluster to innovate, but when a sufficiently large cluster is created their rate of innovation is higher than that of the *traditional firms*.

All the figures show the evolution over time of average results for 10 runs. We have initially considered a general case with default values for the main parameters (IPR = 8; smell-radius = 15; inn-firm-proportion = 50). Figure 3 shows that the innovative firms stay in clusters more than the traditional ones and this happens before for innovative firms than for the others. Figure 4 displays that innovative firms tend to stay in cluster with firms of their same type more than the traditional ones. Figures 5 and 6 show that high-tech firms produce more pieces of knowledge

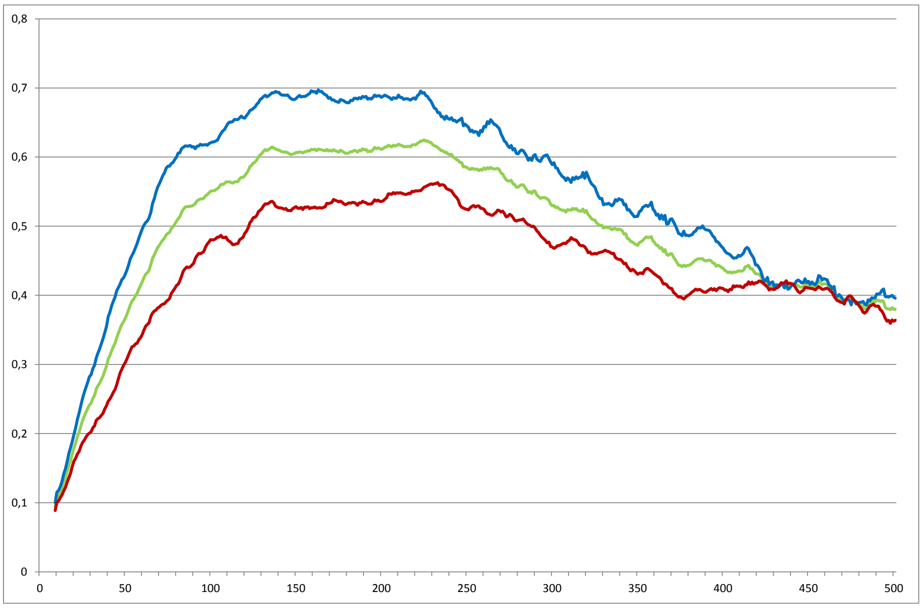


Figure 3: Proportion of firms in cluster (blue: innovative; red: traditional; green: overall). IPR = 8; smell-radius = 15; inn-firm-proportion = 50.

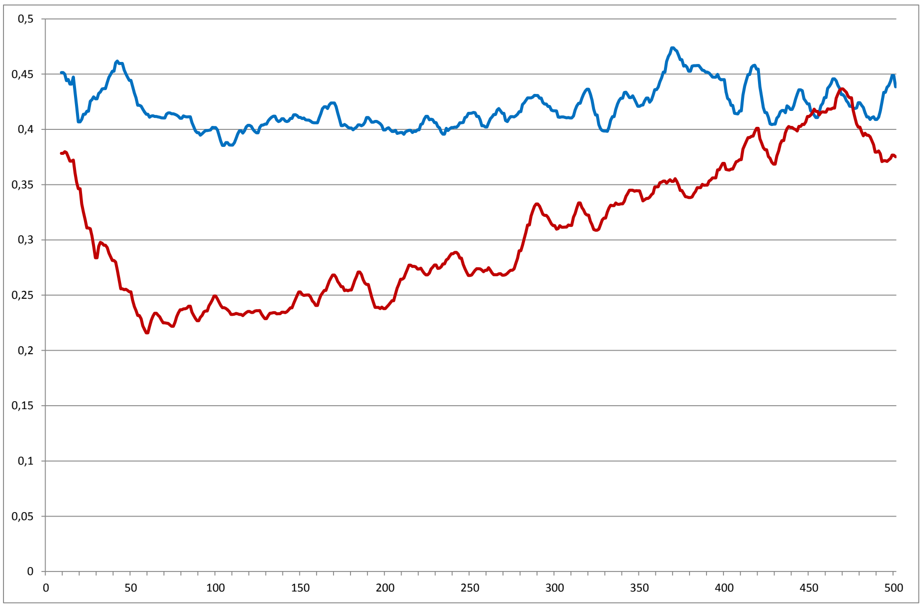


Figure 4: Proportion of links between firms of the same type (blue: innovative; red: traditional). IPR = 8; smell-radius = 15; inn-firm-proportion = 50.

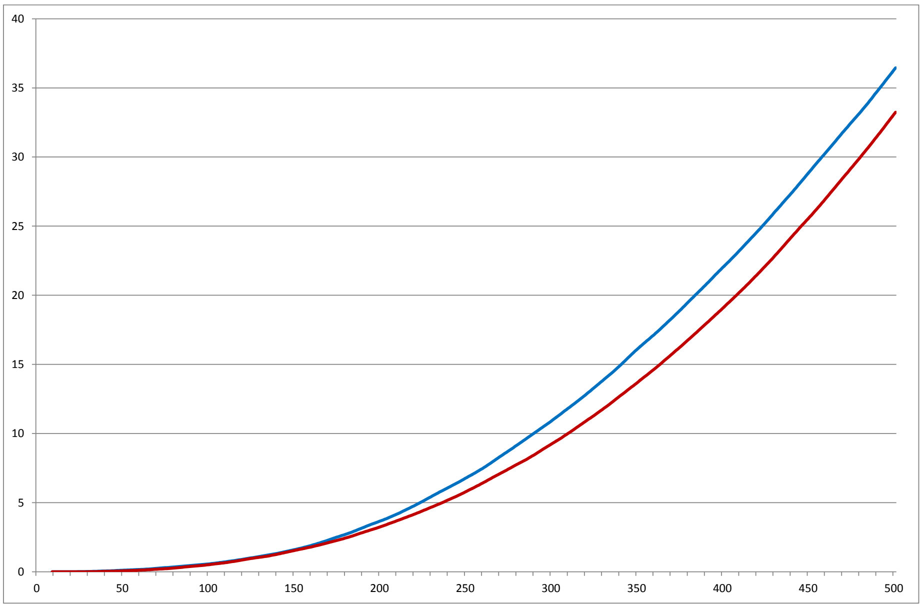


Figure 5: Average knowledge produced (blue: innovative; red: traditional). IPR = 8; smell-radius = 15; inn-firm-proportion = 50.

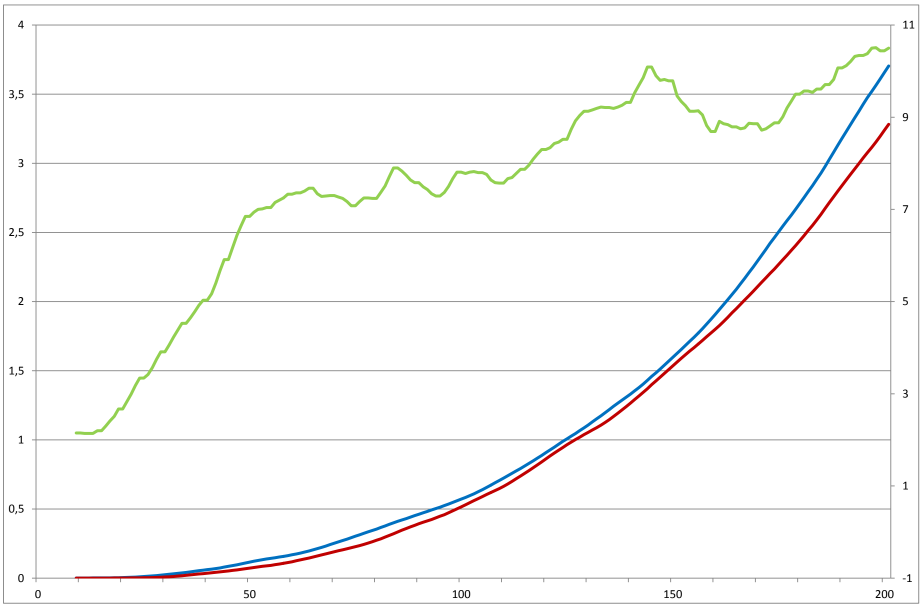


Figure 6: Average knowledge produced (blue: innovative; red: traditional) and largest component dimension (green) in the first 200 ticks. IPR = 8; smell-radius = 15; inn-firm-proportion = 50.

and this effect is particularly strong when a giant component appears (in the second figure you can see that the trend of the blue line changes near the peak of the green line).

The simulations results depend on some parameters: the proportion of innovative firms, the IPRs regime and the area within which the firms can search for other pieces of knowledge (smell-radius). We preset the results of the tests done with different values of these parameters in appendix B.

The tests done with different values of IPR regimes show that for low levels of the parameter the clusters arise in the model after a shorter time (figs. 8 and 9), however no other significant effects are highlighted.

Figures 10 and 11 show that the higher the innovative firms proportion, the more the high-tech firms stay in clusters. The effect is the opposite for the traditional firms, but they produce more new knowledge when the innovative firms proportion grows. Moreover, the dimension of the largest component increases with this parameter (fig. 12).

For low levels of smell-radius parameter traditional firms do not stay in clusters, while the innovative ones are not able to exploit clusters, in which they however are, in order to innovate (figs. 13 and 14). The dimension of the giant component grows with the value of this parameter as shown by fig. 15.

In conclusion, high-tech firms produce more innovation than the others if they are in clusters and can access to enough knowledge. However, the creation and the effectiveness of clusters depend mainly on the dimension of the searching area.

6. Future improvements

Despite the model here described shows some quite interesting results and offers some nice ideas it is however only a rough and green attempt that have to be better implemented under many aspects and parts.

An important step forward could be achieved separating the geographical dimension from the relational one. Indeed, this could allow to understand better the dynamic of the model. Additionally, these two dimensions must be summarised in a network structure that has to be better defined and implemented. The weights of the links in this network will depend on both the geographical closeness and the relational closeness. Once we have such a structure, we can use it to better measure the dynamic of the system and the effect that different levels of the parameters have on the relevant variables. In particular an idea could be to define clusters as K-cores of the network for different values of K. This different definition of the network can also help us to solve some of the questions remain without a clear answer.

Another improvement of primary importance is to design of a different procedure for the demand that considers different consumers' tastes, since the current one is truly implausible.

Moreover, we could introduce some elements useful to make the model a sort of "policy laboratory". First of all, different sources of knowledge – like universities or public structures – can be introduced, also considering that the recent literature has highlighted how they are not the R&D investments per se to be the relevant factors of innovation, but they must be combined with a right environment to show their effectiveness. We could also think to implement firms' knowledge management strategies different from the IPRs one, that is the only possible by the moment, so to simulate different IPRs institutional regimes. In this last case it could be interesting to let the firms free to choose wether to be open or closed to collaboration, maybe learning from their past performances.

A. Technical description and main design solutions

In order to make the model description more understandable and complete, and thereby making it more easily reproducible, we add in appendix some technicalities and design details. To do so, we will follow the so called ODD (Overview, Design concepts and Details) protocol, published for the first time in 2006 and nowadays considered a standard for the published descriptions of Agent-Based Models (ABMs) [Grimm et al., 2010; Railsback and Grimm, 2012].

Since many elements of the protocol were already being widely clarified above, we focus only on the piece of the model still not sufficiently detailed. Where needed we will write some chunks of pseudo-code and we will also use this section as a sort of documentation of the code implemented in *NetLogo* v. 5.1 and available at https://bitbucket.org/nessuno/knowledge-clusters-and-multidimensional-proximity.

A.1. Entities, state variables and scales

The simulated world is a grid of 198×198 and is wrapped both horizontally and vertically, that is a torus. The technological or characteristic space is a ring with 10 blocks one for each technological sector: since there are 100 possible pieces of knowledge in each sector, it is as if each firm had a dresser with 1000 drawers.

A.1.1. Demand side ($\rightarrow$ `movement.nls`)

As we have said, some patches (put on a grid) represents a consumer or a group of consumers that buy the products of the firms. This procedure approximates the "real world" in an extremely simplified way so that is more simple to explain the mechanism as if the firms decide to whom to sell to. Looking at it from this point of view we will see:

1. That a firm, randomly chosen,[8] will chose the consumer with the maximum amount of money between all the consumers that are not further away than 5 patches.

2. The firm will move in the direction of the so chosen consumer, running a distance equal to a fraction of the total distance between them depending on the quantity of money this consumer has, compared with the money the consumer that owns more money has.

3. The so chosen consumer will give the firm in question a part of his/her money, depending on the quantity of knowledge that the firm has.

[8] Each time the order of the firm changes.

4. At each *tick* the stock of money of each consumer ($€_t$) will be increased by $e^{-€_t}$ so that, if the consumer has not spent its money, we have a less than proportional increasing function.

In this way we try to simulate the idea that the firms prefer to stay in a place with a high demand level and that the consumers prefer the variety but have also a decreasing marginal demand.

A.1.2. Independent variables (→ `variables.nls`)

`IPR-strength`
: It indicates the strength of IPRs regime, that is how many `ticks` have to happen before an IPR on a new piece of knowledge expires.

`smell-radius`
: It is the radius of the "smell" spread. It determines the firm's neighbourhood, from which it can get knowledge and in which it checks its profits level.

`innovative-firms-proportion`
: It indicates proportion of innovative firms on the total number of firms.

`time`
: It is the number of `ticks` after which the simulation ends.

A.1.3. Global variables (→ `variables.nls`)

`technology-groups-num`
: It is the number of technological classes in which the firms are subdivided. This variable is set at 10 (→ `setup.nls`) and identifies the length of `mass-array`.

`turtles-num`
: It indicates the number of firms in the model.

`diagonal`
: It is only a technical variable useful to store the dimension of the World.

`knowledge-steps`
: This variable is set at `technology-groups-num` $* 100$ (→ `setup.nls`) and it is the number of possible different pieces of knowledge within an industrial sector. It identifies the length of `knowledge` and `IPR-on-knowledge`.

`constant`
: It is a coefficient of the strength with which a firm attracts another.

`more-best-patch-money`
: It is the maximum value of money among all the consumers.

`max-knowledge-length`
: It is the maximum number of pieces of knowledge that one of the firms currently has.

A.1.4. Firm's variables (→ `variables.nls`)

`localization`

It is the firm's geographical position and its position in the network (Netlogo directly provides it).

`RD-inclination`

It is the percentage of its budget that a firm uses to innovate (when it wants to try to do so). The `traditional-firms` randomly pick their `RD-inclination` within the set $[0.2, 0.8]$ (→ `setup.nls`). While the `innovative-firms` set their `RD-inclination` picking randomly a value within the set $[0.6, 1]$.

`technology`

It is a discrete variable that describes the firm's technological class: it can assume a value greater than 0 and strictly lower than `technology-groups-num`.

`knowledge`

When firms innovate they produce a piece of knowledge. This knowledge is linked to the firm that has made it, but also to the technological group where the firm was when it made the innovation. It is an array of length `knowledge-steps`, whose elements may take the value 0 (no knowledge) or 1 (knowledge). Each technological sector contains 100 pieces of knowledge. At the starting time (→ `setup.nls`), each firm has 5 pieces of knowledge.

The `traditional-firms` put these pieces of knowledge according to a Gaussian distribution with mean in the centre of its technological sector (`technology`) and a variance of 25. So 95% of their initial knowledge is statistically in their technical sector. Instead, the `innovative-firms` have a variance of 75. So 95% of their initial knowledge is either in their sector or in the two adjacent technological sectors.

On each of these initial pieces of knowledge there is an IPR with a random value of `IPR-strength`, as if these pieces of knowledge had been produced at different points in the past. These value are saved in the array `IPR-on-knowledge`

`IPR-on-knowledge`

It is an array of `knowledge-steps` values. The value 0 represents the absence of produced knowledge (its only a potential piece of knowledge for the firm under account). The value 1 means that this piece of knowledge is in the *public domain*. The values [2, `IPR-strength` + 1] tell the time to expiry of the IPRs on this piece of knowledge. Each time that some new knowledge is created, the corresponding value of `IPR-on-knowledge` is set to `IPR-strength` + 1, and then it is updated at each tick.

`budget`

Storage of the profits unused.

`profits`

Each time the firm makes profits.

`profits-threshold`

If the neighbours' average profit is higher than this threshold, then the firm will try to `innovate`, otherwise the `profits` will go into the `budget`.

The `traditional-firms` set their `profits-threshold` as

$$PT_T = \mathcal{U}(30) + 10.$$

While the `innovative-firms` set it as

$$PT_I = \mathcal{U}(30) + 5.$$

`neighbours`

It is the set of firms that are less distant from this firm than the `smell-radius`.

A.1.5. Consumer's variables (→ `variables.nls`)

`money`

These patches that are consumers have a stock of money. They use this stock to buy consumption goods from the firms.

`consumer?`

This variable is equal to 1 if the patch is a consumer. The coordinates of a consumer are multiples of 3 (→ `setup.nls`).

A.2. Process overview and scheduling

The simulation lasts the number of `ticks` indicated by the `time` variable. Each `tick`, the following happens.

A.2.1. Preliminary steps (→ `go.nls`)

1. The variable `more-best-patch-money` is set to the maximum value of money one consumer has, asking each consumer how much money (s)he has.

2. Each firm updates its neighbourhood (`neighbours`), that is an `agentset` of the firms that are within a circumference of radius `smell-radius` around them.

3. Each firm increases its `budget` adding to the previous `budget` stock the new `profits`.
$$K_{t+1} = K_t + \Pi_{t+1}$$

4. Each firm updates its IPRs, that is each element of the `IPR-on-knowledge` array is decreased of 1 provided that it is different from 0 – which would mean that this is a piece of knowledge on which this firm does not know anything about yet – and 1 – which would mean that this is a piece of knowledge which was produced by the firm, but that is now in the *public domain* –. It is important to underline that at the end of this process some pieces of knowledge have gone in the *public domain*.

5. Each firm executes the `set-mass-array` procedure.

A.2.2. Decision to try to innovate and innovation attempt (→ `go.nls`)

1. Each firm executes the `search-neighbours-profits` procedure.

2. Each firm executes the `check-profits-threshold-and-innovate` procedure.

`search-neighbours-profits` (→ `decide-innovation.nls`) Each firm computes the average profit of its neighbours. It does that using an "auxiliary global variable". So it sets `auxiliary-neighbours-profits` to 0 and asks their profits to all the other firms, in order to have something like

$$\overline{\Pi_i} = \frac{1}{\sum_i i} \sum_i \Pi_i, \quad \text{with } i \in \nu$$

where ν is the `neighbours agentset`.[9]

`check-profits-threshold-and-innovate` (→ `decide-innovation.nls`) If the absolute value of the difference between the mean profits in the neighbourhood (`auxiliary-neighbours-profits`) and the firm's `profits` is much higher or much lower than the `profits-threshold` of the firm, it tries to produce a new piece of knowledge, executing the `innovate` procedure.

A.2.3. Innovation attempt (→ `decide-innovation.nls`)

`innovate`(→ `innovation.nls`) Firstly, each firm reduces its `budget` of a certain amount – that is its investment in the R&D activity – depending on its inclination to invest in new knowledge production (`RD-inclination`) as in:

$$K_{t+1} = K_t - K_t * \texttt{RD-inclination}.$$

Then each firm looks around itself search for pieces of available *public knowledge* – that is, pieces of knowledge of the `neighbours` that are fallen in the *public domain* – executing the `public-knowledge-collection` procedure.

Using the pieces of knowledge that are freely available in its neighbourhood (`auxiliary-public-knowledge`), each firm selects the pieces of its interest, i.e. those within a radius of width `knowledge-radius` (ρ) around `mean-internal-knowledge`,[10] and computing the probability of success in the innovation trial as

$$Pr(\mathcal{S}) = \frac{pk_{\varsigma,j} + \sum_i pk^{\tau}_{\varsigma,i}}{2 * \rho} * \texttt{RD-inclination}$$

where the index τ indicates a piece of knowledge in the *public domain*, i is a firm of the `neighbours agentset`, j the firm in question, ς the portion of knowledge that the firm j considers "interesting" and ρ the radius that defines the width of the interesting knowledge portion.

Now, each firm, that wants to innovate, can try to obtain a new piece of knowledge, and it succeeds with probability $Pr(\mathcal{S})$. If it is so, the firm produces a new piece of

[9] Provided that $\nu \neq \emptyset$.

[10] Remembering that the knowledge is defined as circular, so the piece ($\texttt{knowledge} - \texttt{steps} - 1$) is adjacent to the piece 0.

knowledge in a random position within the portion of knowledge of its interest (ς) that is not already occupied by another piece of knowledge that it has just used in the innovation process.[11]

An IPR that will expire after `IPR-strength ticks` is put on this new piece of knowledge.

Each firm that has created a new piece of knowledge executes the following procedure:

1. `mean-knowledge-computation`;
2. `set-mass-array`.

`public-knowledge-collection` (→ `innovation.nls`) Each firm use an "auxiliary global variable" (`auxiliary-public-knowledge`) asking its `neighbours` to collect in it their pieces of knowledge that are in the *public domain*. So, after having re-initialized all the 1000 values of the (`auxiliary-public-knowledge`) array to 0, it puts, in each space of the array, something like

$$PK^{\tau} = \sum_{i} pk^{\tau}_{\varsigma,i},$$

where τ is a piece of knowledge of *public domain* in the firm's interesting sector and i is a firm of the `neighbours agentset`.

`mean-knowledge-computation` (→ `innovation.nls`) Each firm computes the mean value of its internal knowledge (`mean-internal-knowledge`), that is the average position of all the pieces of knowledge it has within its technological sector (`technology`).[12]

`set-mass-array` (→ `innovation.nls`) Each time each firm re-initialises its `mass-array`, so to have an array of length `technology-groups-num` of all 0s.

Then it collects in each element the number of its pieces of knowledge that are in the *public domain* and in the technological sector indicated by the position of current element.

A.2.4. Movements (→ `go.nls`)

Each firm looks around, searching for new opportunities of interaction, executing the `before-move` procedure.

Once every firm has decided where to move, each firm proceeds with the actual movement, executing the `move` procedure.

`before-move` (→ `movement.nls`) Each firm decides where to go in its network, looking for a higher concentration of exploitable knowledge.

[11] That is, is not in the sets of elements $pk_{\varsigma,j}$ and $pk^{\tau}_{\varsigma,i}$.

[12] If it has no knowledge within its technological sector, the value is the centre of the technological sector itself.

The traditional-firms take as influencing firms all its neighbours, while the innovative-firms take all the firms in a circumference of radius the half of the diagonal.[13]

Then all the influencing firms add their mass of attraction for the firm to its mass variable. If the firm is an innovative firm, each influencing firm add to massthe values of their mass-array array in the position of the firm's technology and half of the value of the four adjacent sectors, two on the "right" and two on the "left". If the firm is a traditional firm, each influencing firm add to mass only the values of their mass-array in the position of the firm's technology.

All the influencing firms compute its strength of attraction as

$$\mathcal{F} = \frac{(\frac{\texttt{diagonal}}{2} - \Delta_G)^2 * (\frac{\texttt{techGroupsNum}}{2} - \Delta_T)^2 * \texttt{mass}}{\texttt{c}},$$

with Δ_G the geographical/network distance between the two firms and Δ_T their distance on the technological space, and c the constant.

In the end, each influencing firm compute the strength components on the two Cartesian axes that it gives to the firm and the firm put the difference between them in two temporary variables, xcor2 and ycor2.[14]

move (→ movement.nls) Simply, once that all the firms have executed the before-move procedure, each firm can update its position, so that is as if all the firms decide where to go, knowing nothing of the intentions of others.

Each firm executes the demand procedure to move in the directions of the "consumer" with the higher demand level.

demand (→ movement.nls) The firm checks for the consumer patch, within a distance of 5 around him/her, with the greater amount of money; then (s)he moves in the direction of this "best consumer". The more pieces of knowledge the firm has, the more the patch will pay to him/her. In particular the money transfer will be

$$\Delta_\Pi = \frac{0.2 * \texttt{money} * \sum \texttt{knowledge}}{\texttt{max-knowledge-length}}.$$

While, the "best consumer" strength attraction depends on its amount of money.

Then the firm will update his/her profits as

$$\Pi_{t+1} = \Delta_\Pi + \mathcal{N}(\Pi_t, 1)$$

[13]If the smell-radius is greater than the half of the diagonal all the firms behave as if they were traditional-firms.

[14]What we have called "strength" is more properly an "acceleration", since it is computed by the following «laws of motion»:

$$x = x_0 + a_x * \Delta t^2,$$
$$y = y_0 + a_y * \Delta t^2,$$

with $\Delta t = 1$.

A.3. Design concepts

A.3.1. Emergence

The formation of clusters is, of course, a “built in” result rather than a truly emerging event, but their characteristics, the moment in which we can observe their formation, their evolution and so on are all actual surprising phenomena.

A.3.2. Objectives

The firms try to find places with a big amount of “useful” knowledge and of consumers’ demand. Technically, the other firms or the consumers attract the firm, and is not the firm itself to have an objective function.

A.3.3. Sensing

The reactions of the firms happen in a discrete way: they decide to change their in-equilibrium “standard” behaviour only if they observe some signals: in particular, as we have said, they decide to react when the local conditions of the market are out-of-equilibrium. The firms perceive these situations through a comparison of their `profits` with the average profits of their neighbours.

A.3.4. Interaction

The interaction among firms happens only indirectly, both in terms of knowledge exchanges – this exchange is possible through a movement in the network space – and in terms of competition for the demand market shares – this by a movement in the geographical space.

A.3.5. Stochasticity

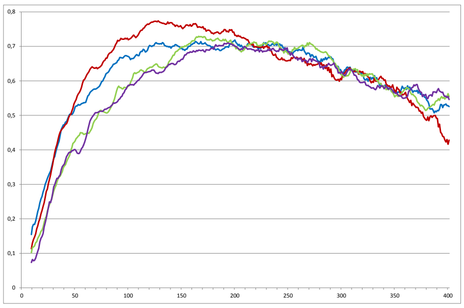
The firms that are in an *out-of-equilibrium* situation will try to innovate, but the success is a stochastic event that depends on the knowledge spillovers and on the R&D investments, and affects the quickness with which in the model new pieces of knowledge are produced – that is not a relevant variable for our purposes.

A.4. Initialization

The initial configuration of the firms in the two-dimensional space representing their geographical/network position is chosen randomly. A cluster configuration was tested without particular effects on the final results and system behaviour. Each firm is different from the others and the heterogeneity in their properties is set in this initial phase. Before each simulation, values of three variables should be initialised:

- `IPR-strength`: it identifies IPR duration and can be chosen in range $[1, 15]$.
- `smell-radius`: it represents the radius of the circle in which firms can access to knowledge and can be chosen in range $[0, 50]$.
- `innovative-firms-proportion`: it is the percentage of `innovative-firms` in the model and can be chosen in range $[0, 100]$.

B. Graphics



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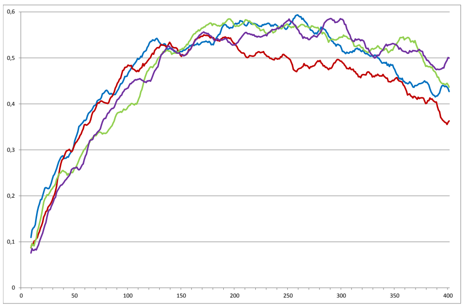

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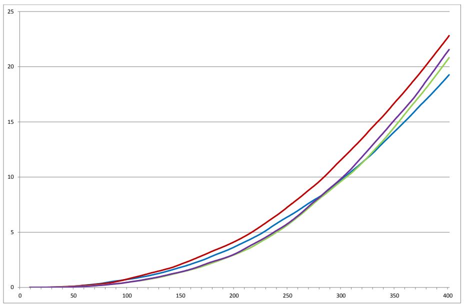

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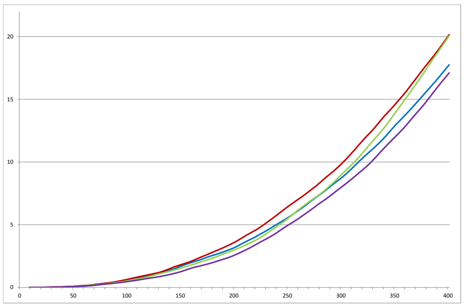


(d)

Figure 7: Outcomes of simulations with different IPR regimes (blue: 1; red: 6; green: 11; purple: 16). smell-radius = 15; inn-firm-proportion = 50. (a) Proportion of innovative f. in cluster. (b) Proportion of traditional f. in cluster. (c) Average knowledge produced by innovative f. (d) Average knowledge produced by traditional f.

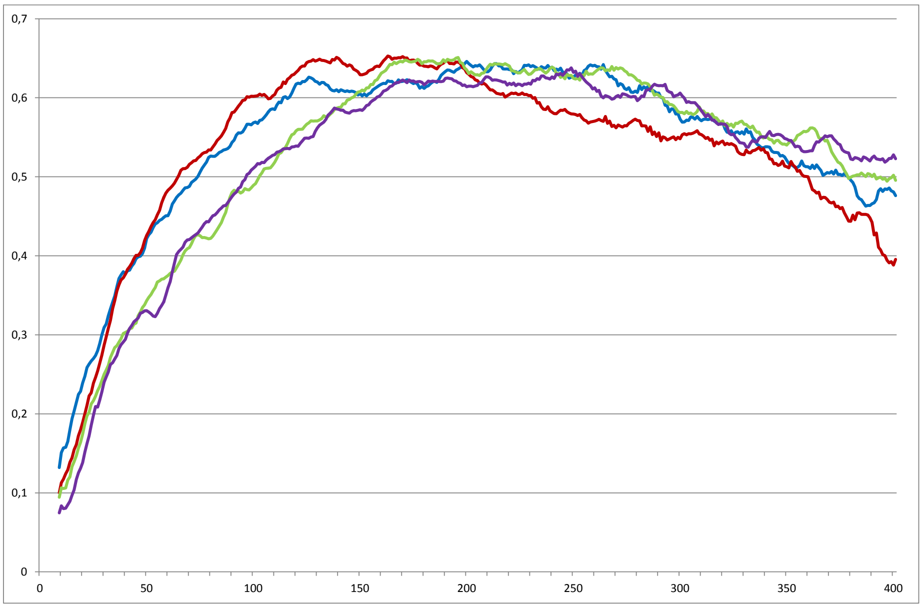


Figure 8: Outcomes of simulations with different IPR regimes (blue: 1; red: 6; green: 11; purple: 16). smell-radius = 15; inn-firm-proportion = 50. Proportion of firms in cluster.

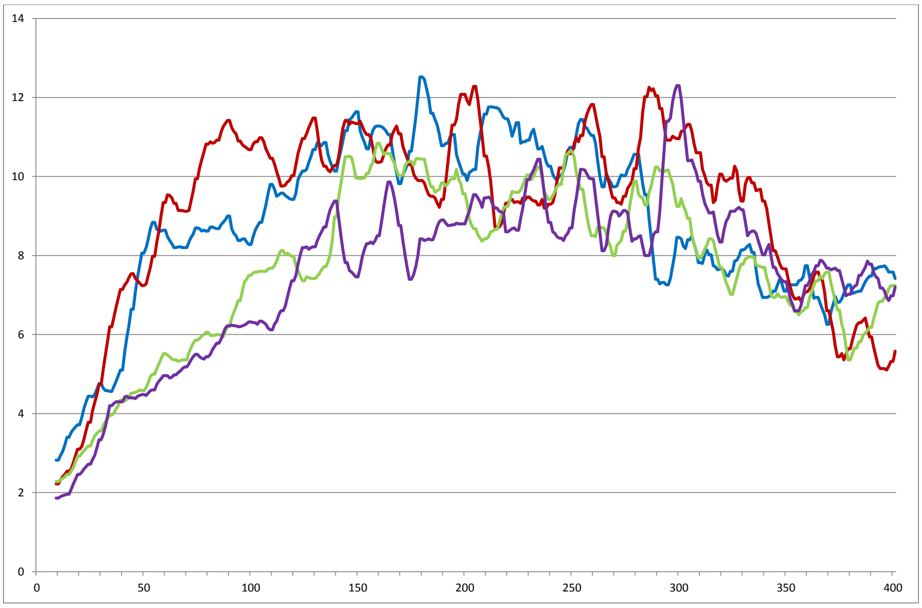
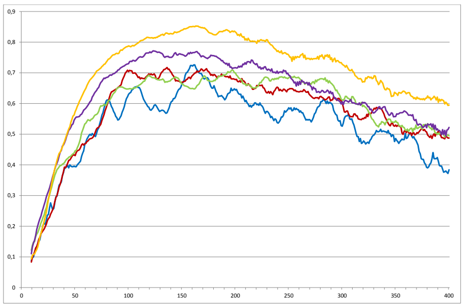


Figure 9: Outcomes of simulations with different IPR regimes (blue: 1; red: 6; green: 11; purple: 16). smell-radius = 15; inn-firm-proportion = 50. Largest component dimension.



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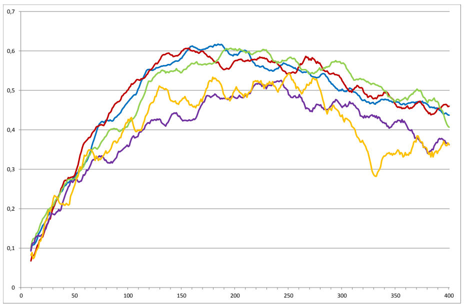

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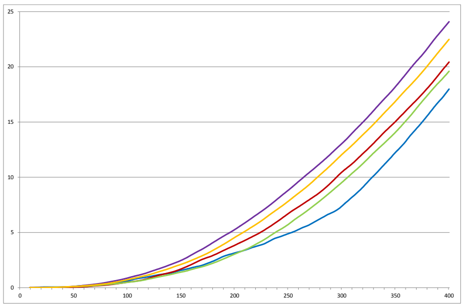

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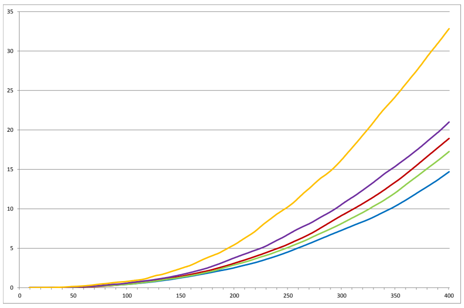


(d)

Figure 10: Outcomes of simulations with different innovative firms proportions (blue: 10; red: 30; green: 50; purple: 70; orange: 90). IPR = 8; smell-radius = 15. (a) Proportion of innovative f. in cluster. (b) Proportion of traditional f. in cluster. (c) Average knowledge produced by innovative f. (d) Average knowledge produced by traditional f.

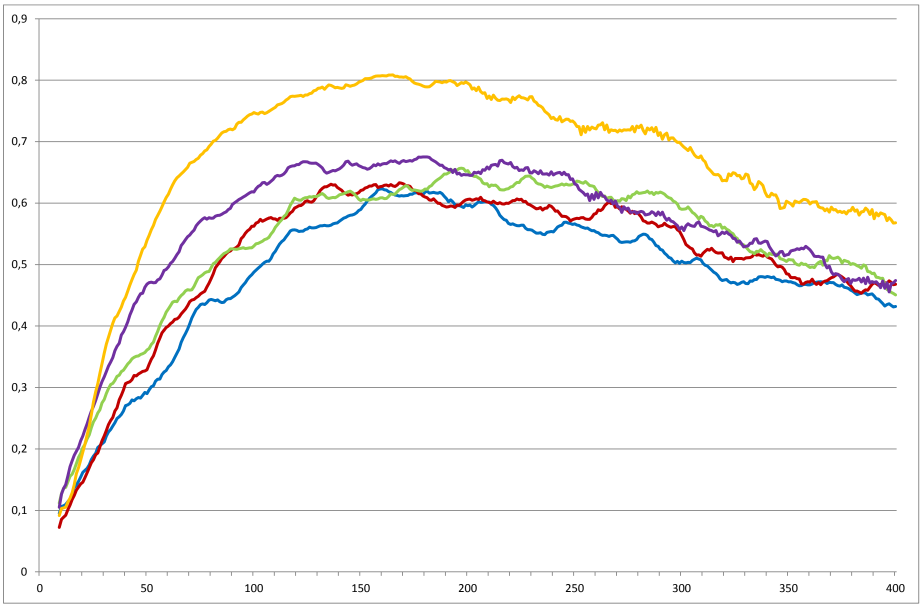


Figure 11: Outcomes of simulations with different innovative firms proportions (blue: 10; red: 30; green: 50; purple: 70; orange: 90). IPR = 8; smell-radius = 15. Proportion of firms in cluster.

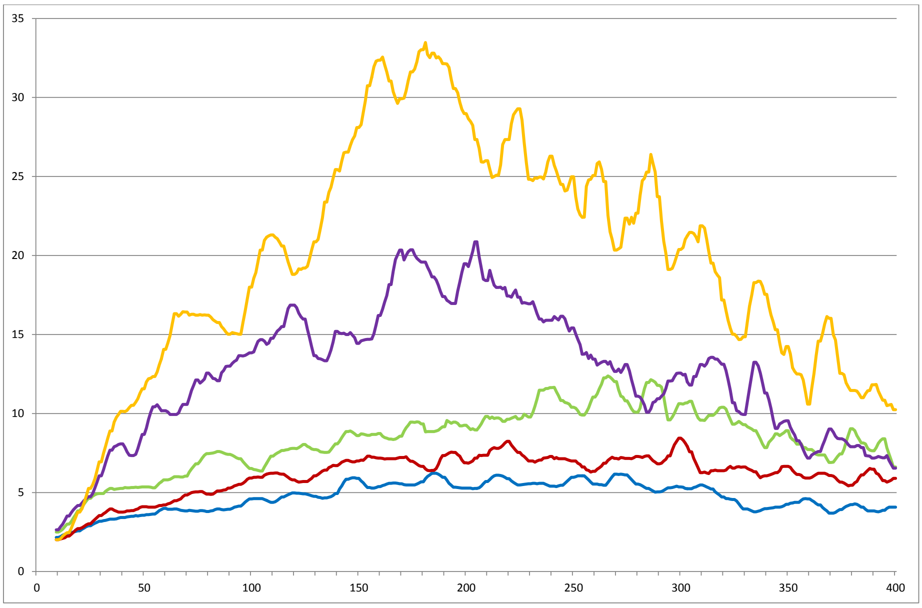
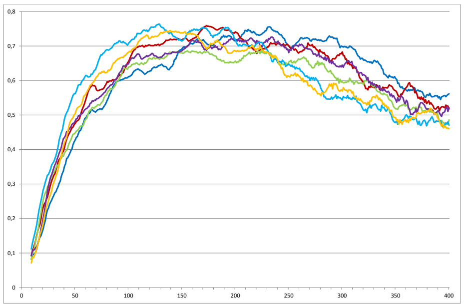
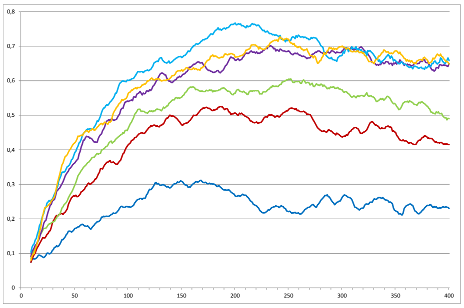


Figure 12: Outcomes of simulations with different innovative firms proportions (blue: 10; red: 30; green: 50; purple: 70; orange: 90). IPR = 8; smell-radius = 15. Largest component dimension.



(a)



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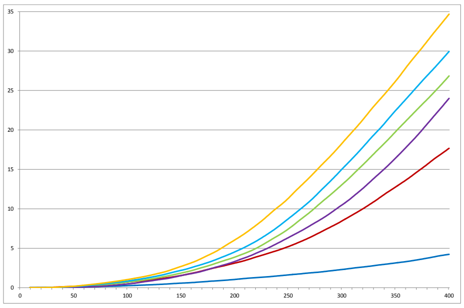

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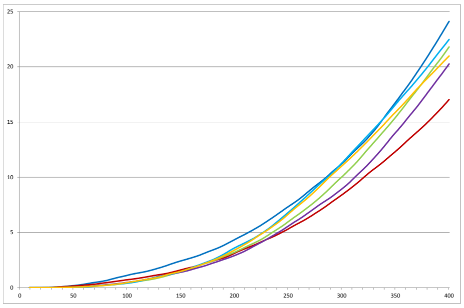


(d)

Figure 13: Outcomes of simulations with different smell-radius levels (blue: 1; red: 11; green: 21; purple: 31; light blue: 41; orange: 51). IPR = 8; inn-firm-proportion = 50. (a) Proportion of innovative f. in cluster. (b) Proportion of traditional f. in cluster. (c) Average knowledge produced by innovative f. (d) Average knowledge produced by traditional f.

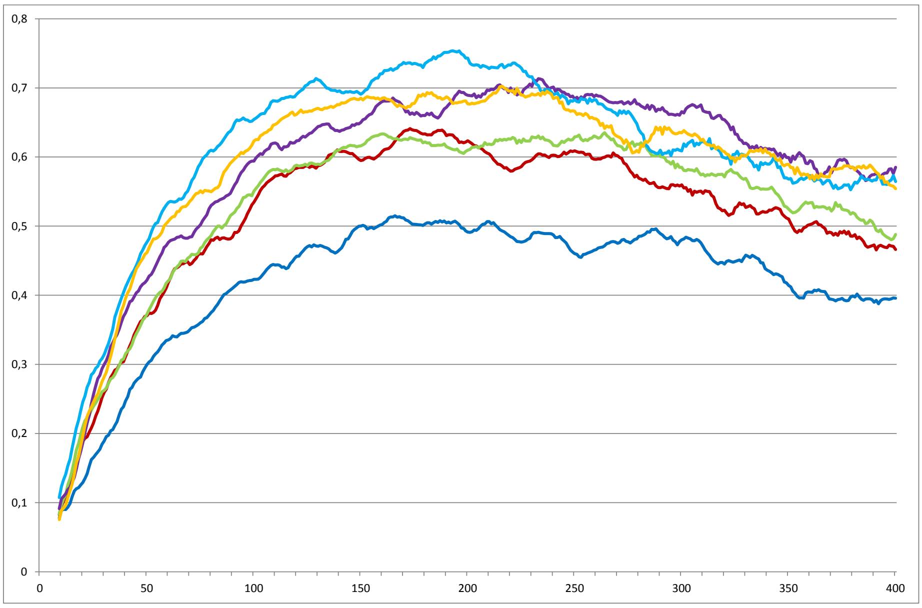


Figure 14: Outcomes of simulations with different smell-radius levels (blue: 1; red: 11; green: 21; purple: 31; light blue: 41; orange: 51). IPR = 8; inn-firm-proportion = 50. Proportion of firms in cluster.

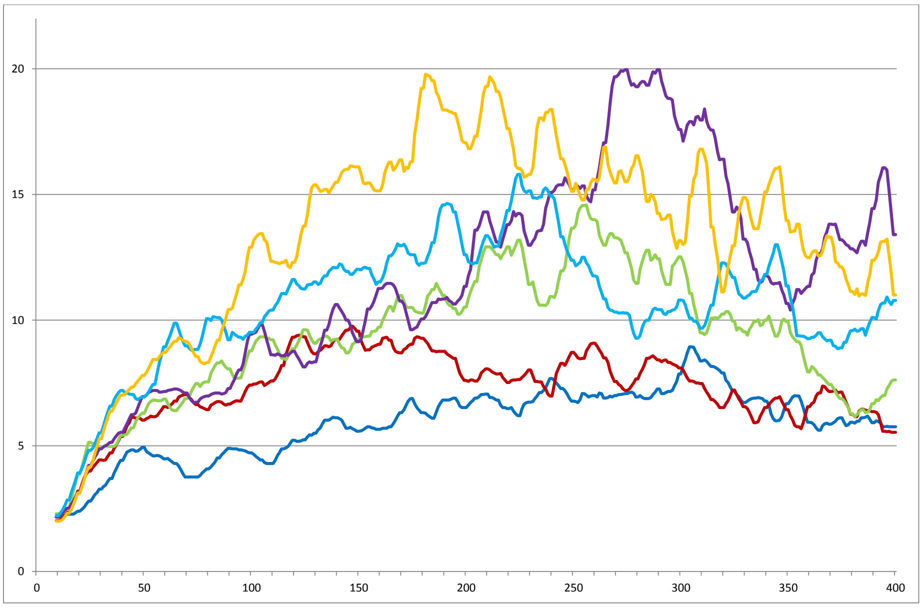
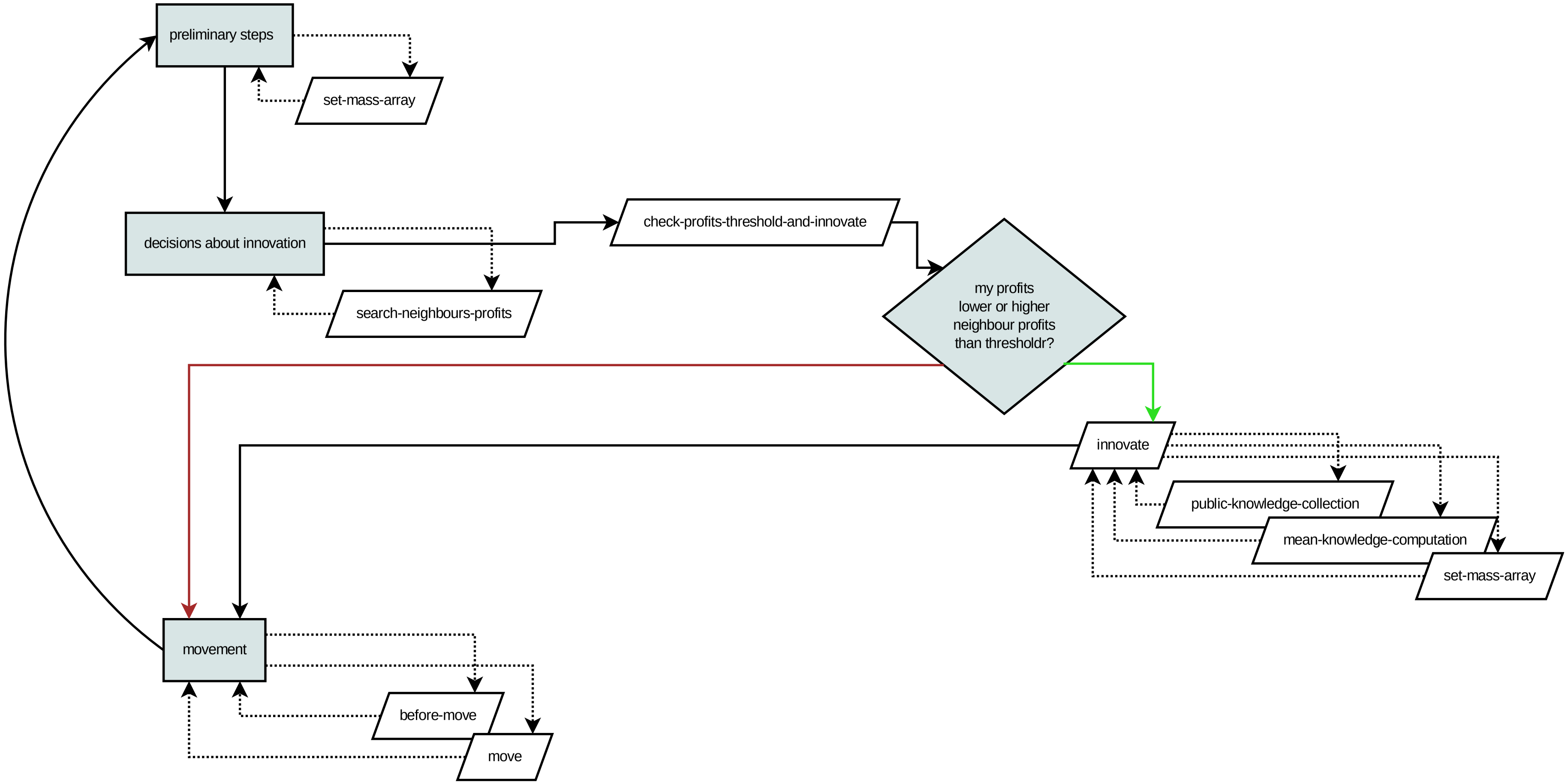


Figure 15: Outcomes of simulations with different smell-radius levels (blue: 1; red: 11; green: 21; purple: 31; light blue: 41; orange: 51). IPR = 8; inn-firm-proportion = 50. Largest component dimension.


preliminary steps
set-mass-array
decisions about innovation
search-neighbours-profits
check-profits-threshold-and-innovate
my profits
lower or higher
neighbour profits
than thresholdr?
innovate
public-knowledge-collection
mean-knowledge-computation
set-mass-array
movement
before-move
move

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