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REMITTANCES AND THE DYNAMICS OF HUMAN CAPITAL IN THE RECIPIENT COUNTRY

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REMITTANCES AND THE DYNAMICS OF HUMAN CAPITAL IN THE RECIPIENT COUNTRY[1]

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Abstract – This paper provides an analysis of the impact of migration and remittances on the inter-generational evolution of human capital in an economy that is characterized by the existence of a poverty trap at a low level of human capital. The analysis is conducted within an overlapping generation model, where parental investment in education are driven by *weakly altruistic* motivations. Remittances boost educational expenditure in recipient households, and they can determine a decisive impact on the long-term dynamics of human capital under favourable assumptions on the wage differential and on migration costs. Under these assumptions, an exogenous probability to migrate represents an equal probability of moving out of the poverty trap, that fades away in the long run, as remittances lead all households to converge towards the equilibrium at a high level of human capital. Although this model does not analyze the general equilibrium effects of remittances – as it is grounded on the independence of households' dynamics – it provides a framework that is open to such an extension, that is called for by the literature on the *Dutch Disease* effects of remittances.

Keywords: Migration, remittances, human capital, poverty trap.

JEL: O15, J24.

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1. Introduction

The critical role that economic theory attributes to human capital in the development process (Romer 1986, Lucas 1988)[2] and the increasing relevance of skilled migration over the last 30 years (Docquier and Marfouk 2005) have both contributed to draw the attention of the literature on the impact of migration on growth via the human capital endowment of the countries of origin. While the recent literature around the so called *brain gain* - or *beneficial brain drain* (Stark and Wang 2002, Beine et al. 2001)– focuses on the impact of the prospect of migration to a high-wage country on the expected returns from educational investments, this paper belongs to the strand of literature that is more concerned with the possible impact remittances on the liquidity constraint that limit these investments.[3]

This paper advances an overlapping generation model to analyze how remittances can influence the intergenerational evolution of human capital in a recipient economy that is characterized by the existence of multiple equilibria in the dynamics of human capital. Thus, it tries to bridge various strands of economic literature, as it draws on the models - originated from the seminal contribution of Galor and Zeira (1993) - that describe the persistence of inequality in the inter-generational dynamics of human capital, and on the models that analyze the impact of remittances on educational expenditure (Rapoport and Docquier 2005, Brown and Poirine 2005). Although it remains outside the scope of the model, we also refer to the literature that suggests the emergence of possible *Dutch Disease* effects arising from remittances, as this may determine non negligible indirect effects on non recipients.

In the model, we focus on the direct effects of remittances upon recipient households, and we derive sufficient conditions that ensure that the increased educational expenditure that is financed with the remittance transfer allows recipient households to escape the poverty trap that characterizes the dynamics of human capital. Under these conditions, a positive probability of migration to a high-wage country modifies the dynamics of the model, as in the long-run all households can move out of the poverty trap and the aggregate distribution of human capital converges towards a stable equilibrium at a high level of human capital. Intuitively, this optimistic prediction on the long-run impact of remittances rests on the existence of a wide wage differential and of low migration costs; when these conditions fail, migration cannot represent a way out of poverty for all. As the direct effects on recipient households may not suffice to lead the poverty trap to fade away, it would be of particular interest to analyze the possible impact on human capital dynamics of the general equilibrium effects of remittances, as there are well-grounded theoretical and empirical reasons to expect them to be significant. The model is designed to allow for an extension in this direction, but the analytical challenges of such an extension move it beyond the scope of this paper. In this respect, its contribution resides in the design of an analytical framework that could allow a future research to bridge the literature on the microeconomic effects of remittances with the one on their macroeconomic impact, as the latter could have a significant influence on the former.

The paper is structured as follows: section 2 reviews the theoretical background of the model, describing – and attempting to defend - its main hypothesis; section 3 develops the overlapping generation model and derives the intergenerational dynamics of human capital when domestic agents cannot opt for migration; section 4 allows for migration, and analyzes the short and long-term impact of remittances on the human capital dynamics of recipient households; both section 3 and 4 rely on the appendix to this paper for the derivation of some of their main results, in order to reduce the burden of the mathematical structure of model on the main text. Section 5 describes the theoretical and empirical justifications for possible extensions of the model that account for the general equilibrium effects of remittances, and section 6 draws the main conclusions of this paper.

[2] The empirical evidence on the impact of human capital on economic growth is, however, controversial (Pritchett 2001).

[3] For an empirical analysis of the impact of remittances on human capital investment see: Cox Edwards and Ureta (2003), Yang (2004), Acosta (2006).

2. Theoretical background and main hypothesis of the model

The model that is presented in this paper builds on Rapoport and Docquier (2005), who sketch a theoretical model to analyze the impact of remittances on the inter-generational transmission of human capital in the presence of poverty traps. Migration - via remittances – can loosen the liquidity constraint that limits the educational investment in recipient households and - assuming a positive intra-generational externality from education – it can produce indirect benefits for non recipient through an increase in the wage rate. The latter is solely determined by the proportion of educated agents in the same generation: once this proportion exceeds a critical threshold, the wage rate moves up from the level w^L to the level w^H.

Rapoport and Docquier (2005) develop an overlapping generation model, where agents live for two periods; in the first period, they receive a parental bequest and may invest in education, while in the second period they inelastically supply their human capital on the labor market and leave a bequest to their children. The model assumes that an indivisible educational investment, with a cost that is normalized to 1, can be financed only through the bequest B that children receive from their altruistic parents, that transfer them a fixed share *b* of their labor income.[4] Agents are *not* utility maximizers, as they realize their educational choice following a simple rule, that is they invest in education as long as the parental transfer is no lower than the cost of education. Denoting with w_{t+1} the wage rate prevailing at time t+1, the bequest left by an economic agent is equal to:

$$B_{t+1} = \begin{cases} b w_{t+1} & \text{if } B_t < 1 \\ b w_{t+1}(1+R) & \text{if } B_t \geq 1 \end{cases}$$

where R is the return to the educational investment. In the absence of migration prospect, Rapoport and Docquier (2005) assume that a poverty trap emerges, as the proportion of educated workers stands below the level that triggers an increase in the wage rate and at the wage rate w^L uneducated workers leave to their children a bequest that does not suffice to cover the cost of education, i.e. $bw^L<1$.[5] Then, the authors introduce the possibility to migrate at time t to an economy where the high wage rate w^H prevails; migration is constrained only by a fixed cost *m* – lower than the cost of education – that has to be paid out of the parental transfer, and a subjective cost such that migrants value a dollar earned abroad just *k* times, $k<1$, a dollar earned at home. As with education, agents follow a simple decision rule: they migrate if they receive a parental transfer in excess of *m* and if the discounted value of the foreign wage w^H exceeds the domestic wage w^L.[6] Once they migrate, the economic agents transfer to their children a share *b* of their foreign income w^H, that suffices to cover the education costs. Thus, migrants' children can afford – via remittances – the cost of the educational investment; if the share of recipient households that get rid of the liquidity constraint that would have been binding in the absence of migration suffices to bring the share of educated workers above the threshold that triggers the increase in the wage rate to w^H, then non recipient households indirectly benefit from remittances. At time t+1, all domestic households will have sufficient resources to invest in education and the economy will move out of the poverty trap to the high wage equilibrium.

The implication of the model is appealing, as remittances can boost educational investment both through a direct income effect on recipient households and through an indirect effect on non recipient that is due to a positive spill-over on the prevailing wage rate. The analysis by Rapoport and Docquier (2005) describes a situation where the parental transfer B belongs to a continuum of values, so that there are some uneducated agent that can afford migration and some that cannot. Still, the dynamics of the bequest B implies that – once the economy is

[4] In Rapoport and Docquier (2005), the parental transfer is coupled with a minimal wage w^M that all agents earn in the first period of their lives, but this can be - for simplicity's sake - set to 0 without modifying any of the implications of their model.

[5] The high wage equilibrium would be sustainable, as the model assumes that $bw^H(1+R) > bw^L(1+R) > 1$, so that educated parents leave to their children a bequest that exceeds 1 even though the prevailing wage rate is w^L.

[6] Suppose that the discounted value of the foreign wage w^H is close to the domestic wage w^L: the migration decision rule implies that agents are willing to pay a positive migration cost out of the parental transfer even though they are nearly indifferent between foreign and domestic employment.

stuck in a poverty trap as it is assumed by the authors - it can assume just two values, namely bw^L and $bw^L(1+R)$. If $bw^L < m < bw^L(1+R)$, then the migration cost can be afforded only by those households that would have anyway bequeathed to their children enough resources to pay for the education cost; in this case, remittances produce neither direct nor indirect effects on educational expenditure. Conversely, if $m < bw^L$, then *all* agents can opt for migration;[7] there is no need to rely on the indirect effects of remittances, as there is *no* non recipient household and the direct effect of remittances alone suffices to bring the economy to the high wage equilibrium.[8]

The main purpose of this paper is to assess the direct effects of remittances in an analytical framework that relaxes some of the assumptions introduced by Rapoport and Docquier (2005), to verify whether their implications still hold. A first departure from Rapoport and Docquier (2005) resides in the introduction of an utility function, so that the decisions regarding education, migration and parental transfer descend from a maximization process of the economic agents, as their prediction of a positive direct effect of remittances on educational expenditure appears to be strongly dependent on their behavioural assumptions. In Rapoport and Docquier (2005), agents invest in education as long as they have enough resources to do so, irrespective of the future returns from education and of the present cost in terms of foregone consumption. Moreover, parents appear to be motivated by a strong descending altruism towards their children when decide to migrate, as they are willing to pay the migration cost m when the discounted value of the foreign wage, kw^H, is just above the domestic wage w^L. This entails that migration produces for them just a direct income cost, with the unique benefit residing in the increased bequest they can confer to their children.

Thus, parents migrate in order to leave children a larger bequest, even though migration may entail just a direct cost for them, and children invest in education as long as they receive a sufficient bequest. Under these assumption, little but a positive direct effect of migration on educational expenditure could have been expected.

We will attempt to analyze the effects of remittances on human capital formation under a set of behavioural assumptions, embedded in an agents' utility functions, that appear to be less inclined towards the prediction of a boost of educational expenditures financed through remittances than those adopted in Rapoport and Docquier (2005). In order to do so, we first move a step back to the literature on the intergenerational transmission of human capital, that provides the broader analytical framework of this paper. The seminal paper by Galor and Zeira (1993) described how persistent inequalities in the distribution of income can arise from an unequal initial distribution of human capital, that persists across generations. The assumption of indivisibilities in the educational investment and of credit market imperfections that determine a higher return when the educational investment is fully financed through parental bequests, give rise to multiple equilibria in the intergenerational dynamics of human capital. Investment in education is thus driven, via bequests, by a descending altruism from the parents to their children. The assumption of descending altruism is maintained by Berti Ceroni (2001), who assumes that parents derive utility from the human capital they bestow to their children, while she takes to the extreme of a self-financing constraint the credit market imperfections assumed by Galor and Zeira (1993). Berti Ceroni (2001) abandons the hypothesis of non-convexities in educational investment, and assumes a concave production function of human capital, that presents the key property of a finite return to educational expenditure even when the latter stands at zero. Without this additional assumption, the decreasing marginal return to educational investment would– even in the presence of a substantial credit market failure - give rise to a unique equilibrium. Both models share a key analytical property, that is a strong segmentation of the dynamics of human capital among households, as the aggregate dynamics of human capital influences neither the return to nor the costs of the educational investments. Thus, the decision problem of each household – and

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[7] Note that the decision rule that underlies migration entails that all domestic agents *do* opt for migration, as $kw^H > w^L$, as otherwise no one would migrate, and thus $kw^H(1+R) > w^L(1+R)$. In a sense, the domestic economy *disappears* for a while, or it is just populated by children who earn the minimal wage w^M.

[8] Indirect effects could still play a role under the assumption that migration is a probabilistic event, subject to an exogenous positive probability. In this case, the progressive increase in the share of educated workers determined by remittances would determine the shift of the prevailing wage rate to w^H, so that there would still be a share of non recipient households that could rely on the indirect effect to get rid of the liquidity constraint on educational investment.

the ensuing human capital dynamics – can be analyzed in isolation, as the influence runs just from the micro to the macro level and not vice versa.

The model assumes that educational expenditures are not driven by the descending altruism of the parents that derive direct utility from the inheritances (Galor and Zeira 1993) or the human capital they confer to their children (Berti Ceroni 2001), but they rather respond to a pension-motivation. As Ray (1998) suggests, developing countries often suffer from the absence or the inadequacy of old-age security schemes, so that children have to shoulder the role of providing support to their parents once they grow old.[9] In the model, parents can choose between securing old-age consumption through an investment in the human capital of their children that entitle them to receive an income transfer from them in the next period, or through the investment on an interest-bearing asset. As in Poirine (1997), the educational investment is framed within an informal family arrangement, that imposes on children the obligation to realize an income transfer to their parents in exchange for the educational expenditures they financed. This arrangement is beneficial for both parents and children as long as the implicit interest on the informal educational loan is at least as high as the rate on the interest-bearing asset, and both are no lower than the rate of return on education (Brown and Poirine 2005). The family arrangement is characterized by what Brown and Poirine (2005) regard as *weak altruism* on the side of the parents, as they require from their children a transfer that leaves them with the same utility they would have enjoyed investing their desired savings level in interest-bearing assets.

A further change from both Rapoport and Docquier (2005) and Galor and Zeira (1993) is the abandonment of an indivisible educational investment, adopting a production function of human capital is a generalization of the one adopted by Berti Ceroni (2001), where human capital is a logarithmic function of the sum of educational expenditure and a component $v>1$, that ensures that all agents have a positive endowment of human capital. We modify this function introducing a fixed cost that gives rise to an initial non-convexity in the returns to educational expenditure. In most developing countries - where basic education is often publicly funded - households have to bear some costs – as those for the enrolment fee or the uniforms - that are fixed at least over the school year, and that are not negligible when compared to household income. As we maintain that credit markets are imperfect and household have to finance their educational expenditures out of retained income (Perotti 1993), the local non-convexities of the human capital production function entail that for low levels of parental income the average return to educational expenditures can fall short of its cost, and no implicit Pareto-improving family arrangement exists. From an analytical standpoint, the hypothesis of initial increasing returns to educational expenditure is critical for the existence of a poverty trap in a model where *weak altruism* is the driving motivation behind educational choices.

Once we have derived the parametric restrictions that ensure that the model is characterized by the emergence of a poverty trap, we assess how the inter-generational dynamics of human capital is affected by the introduction of the possibility to migrate to a high-wage economy.[10] While Rapoport and Docquier (2005) assume that migration is unrestricted, we hypothesize that domestic agents have just a positive probability of been admitted in the destination country. A second difference regards the hypothesis that the migration cost has to be paid out of migrants' *future* income rather than out of the same resources that can be used to finance migrants' education. Their assumption entails that a migration prospect can have an adverse incentive effects on educational expenditure, as some households may decide to cut back educational expenditures in order to cover migration costs.[11] Our alternative assumption rules out such an adverse effects, and it is meant to reflect the idea that would-be migrant

[9] Cigno (2004) advances the idea that inter-generational transfers are governed by an implicit *family constitution* that prescribes to family members their reciprocal obligation; in this framework, transfers to old-age parents correspond to transfers received during the childhood.

[10] The introduction of a migration prospect determines in itself – as predicted by the so called *brain gain* literature (see Commander et al. 2003 for a review) an increase in educational expenditure, but that produces no effect on the long-run dynamics of human capital in the basic framework of the model. As the long standing debate on the *brain gain* versus *brain drain* hypothesis is outside the scope of this thesis, we will not emphasize this aspect of the model.

[11] In a model with more than one child per household, it may emerge that households cut back educational expenditure on some of their children in order to pay migration costs for others.

households can often pool resources other than their retained income in order to finance migration costs.[12]

A third distinctive feature of the model is that it does not represent a one-good economy, but it rather describes a two-sector economy that produces a composite tradable and a non tradable good. This feature, however, remains silent in the specification of the model that is analysed in this paper, as the assumptions on the factor markets suffice to ensure that any imbalance on the goods market is cleared through adjustments of the productive factors rather than through a price change, and that prices do not respond to the aggregate dynamics of human capital. This entails that the extension to a two-good economy does not remove *per se* the complete segmentation of the inter-generational household dynamics of human capital that plays a critical analytical role, as households have no actual interaction via the labor market. Still, this feature is introduced as it allows to broaden the scope of the model to the analysis of the implications for human capital dynamics that may arise from an uneven sectoral impact of remittances. Such an extension of poses severe but unavoidable analytical challenges, as the assessment of indirect effects precisely entails to abandon some of the hypothesis that ensure the decomposability of aggregate evolution of human capital into a collection of independent households dynamics: otherwise, there is no room for any differentiated impact of remittances upon the two sectors.

3. The inter-generational evolution of human capital in the absence of migration

We now develop an overlapping generation model, where agents live for three periods; in period one of their lives they can increase their human capital endowment through the educational investments financed by their parents;[13] in period two they inelastically supply their human capital on the labor market, they consume, realize eventual transfers to the previous generation and to their children, and they can accumulate an interest-bearing asset. In period three they retire and consume out of the present value of their savings and out of the transfers they are eventually entitled to receive from their children; they leave bequest to their descendants.

We assume that educational investments are driven by *weakly altruistic* motivations on the side of the parents, who invest in education only if this increases the present value of children's income over the cost of education (Brown and Poirine 2005). Grown-up children then transfer to their old-age parents an amount that is equal to the educational expenditure they realized, increased according to the prevailing interest rate.[14] Parents obtain the same return from the interest-bearing asset and from educational expenditure, so that it is possible to separate the decision on the size of savings from the one concerning their allocation. We further assume that the credit market is imperfect and we take this assumption to the extreme of a self-financing constraint, so that the educational expenditure cannot exceed the amount that the parents wish to save.

The life time utility function is additively separable in the consumption levels enjoyed in period two and three:[15]

[1] $$U(c_t, c_{t+1}) = u(c_t) + \beta u(c_{t+1})$$

where u is a concave function, β<1 is a discount factor for future utility and c is a composite good, defined as a function of the consumption levels of a tradable good, c^T, and non tradable good, c^N:

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[12] Households could obtain resources outside the nuclear family, migrants could find someone in the destination country that anticipates the necessary funds in exchange for a share of future labor incomes.

[13] We assume that children do not consume, but nothing would change if we introduced an exogenously given positive consumption level in period one of the agents' lives.

[14] In Brown and Poirine (2005), the implicit interest rate on the educational loan can be higher than the market rate, to induce parents to increase their savings; in our model, we will assume a logarithmic utility function, that implies that the optimal savings level is not responsive to change in the interest rate, so that the implicit interest rate coincides with the market rate.

[15] Note that we refer to time t as the time of the adulthood of the agent.

[2] $c_t = \left(c_t^T\right)^{q_T} \left(c_t^N\right)^{(1-q_T)}$, $0 < q_T < 1$.

Both goods are assumed to be non storable. As the economy is assumed to be a price taker on international markets, the domestic currency price of the tradable good, p_t^T, is given by its exogenously given foreign currency price $\overline{p_t^T}$ times the nominal exchange rate E_t. Conversely, the price of the non tradable good p_t^N is endogenously determined. Agents have homothetic preferences with respect to the two consumption goods, so that their respective consumption shares are invariant with agents' incomes. A price index p_t can be defined as a geometric mean of the two prices, with weights equal to their shares in total consumption expenditure:

[3] $p_t = \left(p_t^T\right)^{q_T} \left(p_t^N\right)^{(1-q_T)}$.

Defining Z_t as the expenditure on consumption goods, the consumption level of the composite good c_t can be expressed as:

[4] $c_t = qz_t$, where $q = \left[\left(q^T\right)^{q_T} \left(1-q^T\right)^{(1-q_T)}\right]$ and $z_t = \frac{Z_t}{p_t}$.

Thus, c_t is a linear function of real consumption expenditure z_t. Defining Y_t as the nominal labor income and with T_t the transfers she is eventually obliged to make towards her parent, with $T_t < Y_t$, an agent selects the consumption plan that solves the following maximization problem.

[5] $\max_{S_t} \; u(c_t) + \beta u(c_{t+1})$

$\text{sub } c_t = q\frac{Y_t - T_t - S_t}{p_t}, \; c_{t+1} = q\frac{(1+R)S_t}{p_{t+1}^e}, \; c_t, c_{t+1} \geq 0.$

where p_{t+1}^e is the expected price level for time t+1, R an exogenously given nominal interest rate. With a logarithmic specification of the utility function – that we retain throughout this model, the desired level of savings at time t can be explicitly defined as follows:

[6] $S_t = \frac{\beta}{1+\beta}\left(Y_t - T_t\right) = \frac{\beta}{1+\beta}N_t$

where N_t is the agent's disposable income, that is the labor income minus the eventual transfer towards his parents. Combining condition [6] with [4], we can see that the ratio of the consumption levels enjoyed in period t and t+1 is equal to:

[7] $\frac{c_t}{c_{t+1}} = \frac{1}{\beta\left(1+r^e\right)}$, where $\left(1+r^e\right) = \frac{(1+R)}{\left(1+\pi^e\right)} = \frac{(1+R)p_t}{p_{t+1}^e}$.

3.1 Static equilibrium on the goods and labor market

Labor is assumed to be perfectly mobile among sectors, so that the wage rate per unit of human capital is equal in the domestic tradable and non tradable sectors, while international mobility is costly and subject to exogenous restrictions. In the non tradable sector, human capital in the unique productive factor, and the production is a linear function of the human capital employed in the sector (the assumption of an identity function entails no loss of generality):

[8] $g_t^N = h_t^N$

where h_t^N is the total amount of human capital employed in the non tradable sector. In the tradable sector, both human and physical capital enter a constant return to scale production function. As in Rapoport and Docquier (2004), physical capital is assumed to be internationally mobile and foreign owned; its stock must be such that the profit rate is equal to an exogenously given – and time invariant - rate R. Thus:

[9] $g_t^T = g\left(h_t^T, K_t\right)$, with $h_t^T = \widehat{h_t} - h_t^N$, $\widehat{h_t} = \int h_t b\left(h_t\right) dh_t$ and $\left(\frac{\partial g\left(h_t^T, K_t\right)}{\partial K_t}\right) = R.$

where $b(h_t)$ is the distribution function of human capital that is domestically employed at time t. The profit rate R and the domestic price of the tradable good p^T fully determine the wage rate in the tradable sector, W_t. As labor is intersectorally mobile, this needs to equal the wage rate in the non tradable sector, that by [7] is simply equal to p_t^N. This implies that the equilibrium on the labor market requires that:

[10] $a_t = \frac{p_t^T}{p_t^N} = \left[\rho\left(R\right)\right]^{-1}$, where $\rho\left(R\right) = \frac{\partial g\left(h_t^T, K_t\right)}{\partial h_t^T}$ sub $\frac{\partial g\left(h_t^T, K_t\right)}{\partial K_t} = R$.

The ratio of the price of the tradable over the non tradable good, a_t, must be equal to the inverse of the marginal productivity of human capital. As g is linearly homogenous, the latter is fully determined by R, the exogenously given profit rate, so that we can express it as a function ρ of the profit rate R. Combining [10] with [3], we have that:

[11] $p_t = E\overline{p_t^T}\left[\rho\left(R\right)\right]^{1-q_T}$.

Combining condition [10] with the definition of the price index p_t, we have that the real wage w_t is equal to:

[12] $w_t = \frac{W_t}{p_t} = \frac{p_t^N}{E\overline{p_t^T}\left[\rho\left(R\right)\right]^{1-q_T}} = \frac{E\overline{p_t^T}\rho\left(R\right)}{E\overline{p_t^T}\left[\rho\left(R\right)\right]^{1-q_T}} = \left[\rho\left(R\right)\right]^{q_T}$.

Thus, the labor market equilibrium determines the equilibrium wage rate and the relative intersectoral price, so that market clearing on the goods market must be achieved through adjustments in consumption and production, via the sectoral allocation of labor. The appendix A1 shows that an allocation of labor that ensures the equilibrium on the goods market always exists, so that we can analyze the model regarding w_t as the equilibrium wage rate.

3.2 The dynamics of the human capital endowment

As in Berti Ceroni (2001), we assume that all agents are endowed with a minimal level of human capital μ even if their parents have not devoted any resource to educational investment. The labor income for an agent is thus given by her human capital endowment h_t times the prevailing nominal wage rate W_t:

[13] $Y_t = W_t h_t$, with $h_t \geq \mu$.

Human capital for the t+1 generation is formed through educational expenditure F_t, according to the following production function:

[14] $h_{t+1} = \begin{cases} \ln\left[(f_t - \theta) + \nu\right] & \text{if } f_t > \theta \\ \mu & \text{otherwise} \end{cases}$, where θ >0, $\nu = e^{\mu}$

f_t represents the expenditure in education F_t deflated by the price index p_t. We assume the existence of fixed costs in educational investments, that are set at a positive level θ. If parents do not invest in education, the children get the minimal endowment in human capital μ. If they do invest, as in Brown and Poirine (2005) they are then entitled to receive in period t+1 a transfer from their children equal to the educational expenditure F_t plus a remuneration equal to the interest rate R. We are thus abstracting from the issue, analysed in Cigno (2004), that agents could not comply with the intergenerational obligations arising from past expenditure in education. Labelling this transfer as T_{t+1}, we have that:

[15] $T_{t+1} = (1+R)F_t$.

The motivation behind educational expenditures are not selfish, as [15] implies that children entirely benefit from any increase in their labor income above the current value of past educational expenditures, as the parents' return from educational investment is the same they would get from the holdings of an interest-bearing asset. The existence of a fixed cost θ may give rise to an equilibrium at a low level of human capital, if those who are endowed with an human capital μ do not have an incentive to invest in the education of their children. Indeed, the fixed cost θ entails that only those parents whose labor income, i.e. human capital, exceeds a critical threshold will invest in education. As the marginal impact of educational expenditure on human capital is finite even at a zero expenditure, a corner solution, with all income devoted to consumption, may be optimal for low levels of parental income, i.e. human capital. In the absence of fixed costs, the distribution of human capital would converge to a unique equilibrium, as poorly endowed agents would enjoy higher return to education.
We have seen that the level of parental savings is solely determined by their disposable income, while its allocation depends on the relative profitability of monetary savings and educational expenditure. As we have assumed that the latter is bounded by a self-financing constraint, we have that:

[16] $f_t \leq \frac{S_t}{p_t} = s_t = \frac{\beta}{1+\beta}\left(\frac{Y_t - T_t}{p_t}\right) = \frac{\beta}{1+\beta}(y_t - t_t)$

The above condition implies that the educational investment is constrained by the parents' disposable income, and could thus be suboptimal from a child stand-point. Relations [14] and [15] imply that the optimal investment in education for the child is equal to:

[17] $f_t^* = \frac{w_{t+1}^e}{1 + r_{t+1}^e} + \theta - \nu$,

where w_{t+1}^e and r_{t+1}^e are the expected values for time t+1 of the real wage and of the real interest rate respectively. As the final interest of this model resides in assessing the impact of migration prospects and of migrants' remittances upon human capital formation, we introduce an assumption that greatly simplifies the analysis of the model dynamics if the possibility to migrate is banned and all the labor force is domestically employed. We assume that the foreign price of the tradable good and the profit rate are time-invariant. Given the macroeconomic structure of the model, this rules out any change in the real exchange rate and in the wage rate per unit of human capital. The unique source of divergence between current and expected level of the variables resides in the introduction of a positive probability to migrate to a higher wage country. Thus, in the absence of migration prospects we have that $w_{t+1}^e = w_t = w$, and $r^e = r = R$, as there is no price inflation. The optimal level of human capital is thus given by:

[18] $h^* = \ln\left(\frac{w}{1+R}\right)$,

The value of educational expenditure defined by [17] is the one that ensures the equality of the expected marginal benefit and marginal cost of education for the child. The actual educational investment is deemed to fall short of f^* as long as $s_t < f^*$. Any educational investment that exceeds θ but falls short of f^* has a marginal return that exceeds its marginal cost; still, a desired saving level that exceeds θ needs not to be devoted to educational expenditure, as the fixed costs entailed by human capital formation imply that the average return from education can stand below its average cost. Indeed, weakly altruistic parents invest in education only if their desires real savings level s_t, defined by [6] is such that:

[19] $W\ln\left(s_t - \theta + \nu\right) - \left(1+R\right)s_t > W\mu$,

where the right hand side of [19] is the labor income in the case of no investment in education. Condition [19] implicitly identifies two critical values of human capital, that we label respectively h^0 and h^1: the first one is the level of human capital that determines a disposable income equal to $W\mu$, so that h^0 is the minimal endowment of human capital that can be profitably bestowed to generation t+1. On the other hand, h^1 is the level of parental human capital that determines a desired level of savings that suffices to finance the educational expenditure required to form h^0. From [19], we have that h^0 can be defined as follows:

[20] h^0 s.t. $\left.\frac{e^{h_{t+1}} + \theta - \nu}{h_{t+1} - \mu}\right|_{h_{t+1}=h^0} = \frac{w}{(1+R)}$ and $\left.e^{h_{t+1}}\right|_{h_{t+1}=h^0} < \frac{w}{(1+R)}$

No child will ever be endowed with a human capital that is lower than h^0 if her parents do invest in education. Going a step back, we have that just those agents with a human capital in excess of h^0 will have some transfer obligations towards their parents, so that the real disposable income n of an agent who is endowed with an human capital equal to h_t is:

[21] $n(h_t) = \begin{cases} \left(wh_t - (1+R)\left(e^{h_t} + \theta - \nu\right)\right) & \text{if } h_t > h^0 \\ w\mu & \text{otherwise} \end{cases}$

From [20] and [21], we can derive the level of parental human capital h^1, such that parents have a desired savings level that allow them to bestow to their children the human capital h^0:

[22] h^1 s.t. $\left.\frac{\beta}{1+\beta}\left[wh_t - (1+R)\left(e^{h_t} + \theta - \nu\right)\right]\right|_{h_t=h^1} = \left(e^{h^0} + \theta - \nu\right)$

Finally, we can define a level of h_t, labelled h^2, such that parents endowed h^2 have a desired savings level that allows them to finance the educational expenditure that is optimal for their children:

[23] h^2 s.t. $\left.\frac{\beta}{1+\beta}\left[wh_t - (1+R)\left(e^{h_t} + \theta - \nu\right)\right]\right|_{h_t=h^2} = \left(\frac{w}{1+R} + \theta - \nu\right)$

Using conditions [20] to [23], we can now describe the dynamic evolution of human capital, as we know the disposable income of an agent at time t, and hence her desired savings level, and the cut-off points that describe the point were agents start to invest in education and where they reach the optimal investment. We have that:

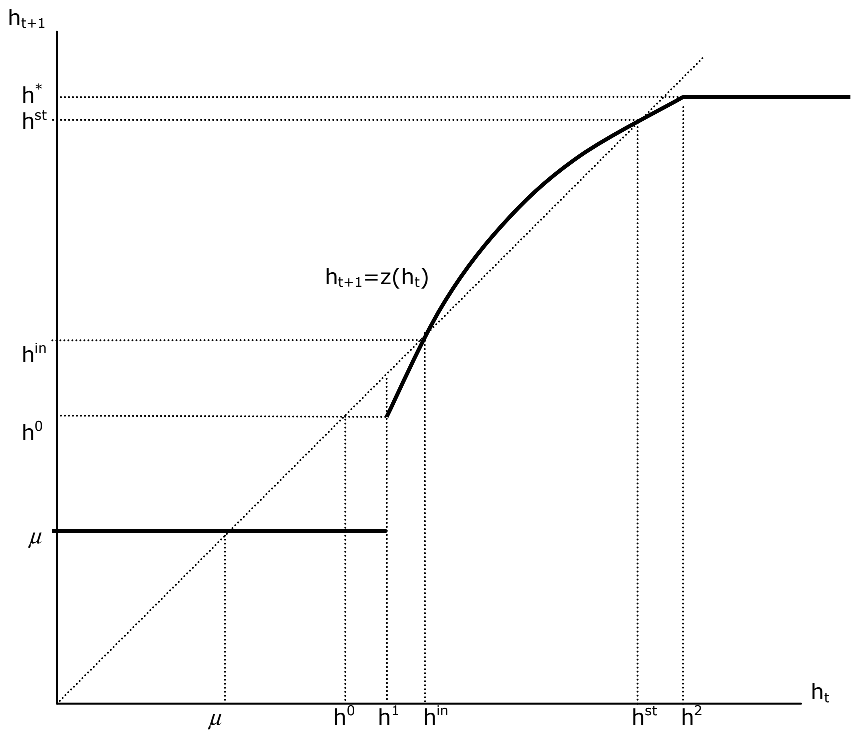
$$[24] \qquad h_{t+1} = z(h_t) = \begin{cases} \mu & \text{if } h_t < h^1 \\ \ln\left(\frac{\beta}{1+\beta}\left[wh_t - (1+R)\left(e^{h_t} + \theta - \nu\right)\right] - \theta + \nu\right) & \text{if } h^1 \le h_t \le h^2 \\ \ln\left(\frac{w}{1+R}\right) & \text{if } h_t \ge h^2 \end{cases}$$

Under the following restrictions – that are derived in the appendix A2 and A3 - on the level of the parameters θ and ν:

$$[25] \qquad \theta^1 = \nu + \frac{\beta}{1+\beta} w \ln(\nu) - \nu^{\frac{\beta(1+R)+(1+\beta)}{1+\beta}} < \theta < \nu + \frac{w}{\gamma}\left[\ln\left(\frac{w}{\gamma}\right) - 1\right] = \theta^2, \;\; \nu \neq \nu^M, \text{ where } \gamma = \frac{\beta(1+R)+(1+\beta)}{\beta}$$

the dynamics of human capital across generations described by $h_{t+1}=z(h_t)$ has a qualitative behaviour that can be represented by the following figure:

Figure 1. Dynamic evolution of human capital in the (h_t, h_{t+1}) space.



The function z presents a discontinuity for $h_t=h^1$ that is due to the fixed cost that is assumed to be associated to the investment in education; the closer is the parameter θ to its lower bound θ^1, the smaller is this discontinuity, as h^0 gets closer to μ. The system presents three equilibria, with two of them being locally stable and one unstable. As long as $\theta > \theta^1$, educational expenditure is not profitable at $h_t= \mu$, so that the minimal endowment of human capital can persist across generations. This equilibrium is locally stable, as the function z is flat for $h_t < h^1$. The restriction $\theta < \theta^2$ ensures that there is a second, locally unstable, equilibrium for $h_t=h^{in}$ and a third, stable, equilibrium for $h_t =h^{st} <h^*$.[16]

[16] If the restrictions on the parameters θ and ν introduced by [25] are not satisfied, then the system converges towards a unique stable equilibrium from any initial endowment. More specifically, whenever $\theta < \theta^1$, the unique equilibrium is found at h^{st}, as fixed costs are so low that educational expenditure is profitable also at $h_t = \mu$, and there is a convergence towards an equilibrium at a high level of human capital as poorly endowed households enjoy higher returns from educational investments. Conversely, if $\theta > \theta^2$, then fixed costs are so high that no level of human capital except μ can be sustained across generations, and all households converge towards the minimal endowment of human capital.

The unstable equilibrium h^{in} plays a critical role, as it divides the attraction basins of the two locally stable equilibrium. The long-run evolution of human capital within an household is entirely determined by the position of the initial endowment of human capital with respect to h^{in}. If this stands at the right of h^{in}, then the household will increase its human capital over time, converging towards h^{st}, while if its initial endowment falls short of h^{in}, the human capital will decline across generations and in a finite time it will reach the minimal level μ. Credit market failures and the existence of a fixed cost associated to educational investment entail that inequalities in the distribution of human capital can persist over time, as poor households cannot afford to invest in the education of their children. The appendix A4 demonstrates that h^{in} increases – and hence the dimension of the attraction basin of μ widens - as the fixed cost for education θ gets higher. Intuitively, the higher the educational cost, the harder it is to have enough resources to finance an educational investment that suffices to leave the poverty trap.

4. The impact of migration and remittances on human capital dynamics

As Berti Ceroni (2001) observes, the existence of a poverty trap may be fragile once we introduce uncertainty in the model, in the form of an exogenous shock that can move households across the critical threshold h^{in}; in this case, the long run distribution of human capital could still converge to a unique equilibrium. In our setting, such an exogenous shock is represented by a positive probability λ to migrate to a high-wage country at the beginning of the second period. While the domestic mobility of labor across sectors is costless, migration is assumed to entail a fixed real cost τ, that has to be paid out of migrant's labor income;[17] migrants return home in period three, as this behavior can be justified as the home country is characterized by a lower price level (see below). When abroad, migrants send back remittances, that are made up of distinct components: the transfer towards their parents to pay back past educational expenditure and transfer towards their children to finance their human capital formation.

We hypothesize that the destination country is identical to the home country, with the exception of a technological superiority in the tradable sector, that determines a higher real wage rate than the one prevailing in the home country. We denote with the superscript f the variables and functions that refer to the foreign country; the technological superiority of the foreign country thus entails that the productivity of human capital in the tradable sector is higher abroad for any level of the profit rate R, that is $\rho^f(R) > \rho(R)$. The equilibrium on the foreign labor market entails that the wage rate is equal to:

$$[26] \qquad W^f = \overline{p_T}\left[\rho^f(R)\right]$$

By analogy with the domestic one, the foreign price level is given by:

$$[27] \qquad p^f = \overline{p_T}\left[\rho^f(R)\right]^{1-q_T}$$

Imagine that an agent with h_t units of human capital is offered the chance to migrate. The decision to migrate is taken comparing the life-time utility in the alternative scenarios of domestic and foreign employment. These depends on the disposable income the agent can obtain in the two countries and on the existing price differential. This follows straight from the wage differential between the home and the foreign country, and it is relevant in the choice of the would-be migrant as their consumption in period 2 takes place in the country of employment. As migration is costly, this may not be remunerative for low endowments of human capital, as the positive wage differential may be offset by the fixed cost τ. The appendix

[17] Note that we could change the order of the events, that is we could endorse the more realistic assumption that agents have to pay a cost τ in order to get a positive probability λ to be admitted in the foreign country. This would not change the results of the analysis, but it would add an analytical difficulty, as the disposable income of an agent who is domestically employed would not depend only on his human capital endowment, but also on whether he has attempted or not to migrate. A failed, i.e. non successful, attempt to migrate would lower the agent's disposable income and hence his capacity to invest in the education of his child.

A5 shows that an agent who is offered the chance actually migrates only if her human capital endowment is in excess of a critical threshold h^{mig}, that is defined as:

$$[28] \qquad h^{mig} \text{ s.t. } \left.\frac{\left(w^f h_t - \tau d^{-1} - (1+R)\left(e^{h_t} + \theta - \nu\right) d^{-1}\right)}{\left(w h_t - (1+R)\left(e^{h_t} + \theta - \nu\right)\right)}\right|_{h_t = h^{mig}} = d^{\frac{-\beta}{(1+\beta)}}, \text{ where } d = \frac{Ep^f}{p} = \left(\frac{\rho^f(R)}{\rho(R)}\right)^{1-q_T} > 1.$$

The variable d is the real exchange rate faced by the domestic economy, that is greater than one given the technological superiority of the foreign economy. The appendix A5 shows that if the parameters w^f and τ that determine the profitability of the migration venture satisfy the following condition:

$$[29] \qquad \tau < \left[\left(\frac{w^f}{w}\right)^{\frac{1}{q_T}} - \left(\frac{w^f}{w}\right)^{\frac{(1-q_T)}{(1+\beta)q_T}}\right] w\mu$$

then $h^{mig} < \mu$, and all agents are willing to migrate once they are offered the chance. Condition [29] illustrates that, for a given wage differential, the fixed cost that has to be paid to migrate needs not to exceed the threshold implicitly set by [29] for migration to be a profitable option for poorly endowed agents. If [29] is not met, then migration is not viable for those agents that are stuck in the lowest equilibrium, and hence neither migration nor remittances have a direct impact on this poverty trap. If [29] does not hold, then migration *selects positively* with respect to the current human capital endowment, i.e. household income. The higher τ, the higher h^{mig}, that is the threshold level of human capital that is required to regard migration as an option to increase one's own utility. A natural hypothesis is that τ cannot be so high to set the value of h^{mig} above h^*, otherwise migration would be not viable for all domestic agents and the system would not be affected by the introduction of a positive probability λ of migration.

4.1 Migration prospects and the incentives to invest in education

Condition [28] reflects the choice of an economic agent once migration is dependent only on her will, but migration is an uncertain event, as agents are given just a positive probability λ to migrate. As we have modelled human capital as a continuous variable that is remunerated at a constant wage rate, a positive probability to migrate clearly raises the incentives to invest in education, as migration to a high-income destination is a migration towards a country where the skill premium is higher, although some of the empirical evidence suggests that wage dispersion is higher in developing countries (World Bank 1995).[18]
We can recall that the desired size of savings and their allocation between educational expenditure and monetary savings are two distinct problems facing an economic agent; the former is determined by an agent's disposable income, and hence by his endowment of human capital, while the latter depends on the relative profitability of the two available options. A positive probability λ to migrate has no influence on the desired level of savings, but it increases the attractiveness of educational expenditure, as children now have a chance to be employed in a high-wage country once they become adult. Thus, the migration prospect for the generation t+1 has no effect for the households that at time t have a human capital endowment h_t such that $h^1 < h^2$, as they already devote all their savings to human capital formation. A positive λ can influence only those households that at time t would have decided to hold some monetary savings if domestic employment was the unique option, that is the households with a human capital endowment lower than h^1 or higher than h^2. Let us first consider the second case: the households with $h_t > h^2$ increase switch a part of their savings towards educational expenditure, as the optimal level of human capital is increasing in λ as

[18] Such an effect would not occur in a model were educational expenditures are solely driven by an altruistic motivation of the parents that derive utility from the human capital they bestow to their children as in Berti Ceroni (2001).

long as $h^{mig} < h^*$. Once there a positive probability to migrate is introduced, the expected life-time utility from any given level of human capital $h_{t+1} > h^{mig}$ is a weighted average of the life-time utility in case of domestic and foreign employment, $V^d(h_t)$ and $V^f(h_t)$, with weights that coincide with the probability of each scenario. For $h_{t+1} \leq h^{mig}$, the expected life-time utility coincides with the domestic one, as migration is ruled out as a non profitable option. The expected life-time utility $E[V(h_t)]$ is thus given by:

$$[30] \qquad E\left[V(h_t)\right] = \begin{cases} V^d(h_t) & \text{if } h_t \leq h^{mig} \\ (1-\lambda)V^d(h_t) + \lambda V^f(h_t) & \text{if } h_t > h^{mig} \end{cases}$$

The value of h_t that maximizes $E[V(h_t)]$ depends on the exogenous probability λ, so that we label it as $h^*(\lambda)$, and it is defined as follows:

$$[31] \qquad h^*(\lambda) \text{ s.t. } \left.\frac{\partial E\left[V(h_{t+1})\right]}{\partial h_{t+1}}\right|_{h_{t+1}=h^*(\lambda)} = \lambda \frac{\partial V^f(h_{t+1})}{\partial h_{t+1}} + (1-\lambda)\left.\frac{\partial V^d(h_{t+1})}{\partial h_{t+1}}\right|_{h_{t+1}=h^*(\lambda)} = 0 \text{ for } h_{t+1} > h^{mig}$$

For any positive probability of migration, it is immediate to observe that:

$$[32] \qquad \frac{\partial h^*(\lambda)}{\partial \lambda} > 0$$

so that the optimal level of human capital in increasing in probability of migration. All the households with a human capital h_t in excess of h^2 will thus react to the possibility to migrate with an increase in their educational expenditures.

The impact of a positive λ on the households that hold interest-bearing assets at the opposite end of the spectrum of human capital, that is for $h_t < h^1$, depends on the relative position of h^{mig} and h^0. If the fixed cost τ entailed by migration is such that $h^{mig} > h^0$, then households that have a level of human capital h_t lower to h^1 do not modify the allocation of their savings in response to a positive probability to migrate, as they are unable – because of the binding self-financing constraint – to bestow their children with the minimal level of human capital that renders migration profitable. Conversely, if $h^{mig} < h^0$, migration can be a viable option, and it can induce a positive educational expenditure at a level of parental human capital lower than h^1. Further suppose that [29] holds, that is $h^{mig} < \mu$, so that all economic agents have an incentive to migrate;[19] educational expenditure is profitable from the child stand point if she is bestowed with a human capital h_{t+1} in excess of $h^0(\lambda)$ that is defined as the level at which the expected life-time utility coincides with the one of an agent that has received no education:

$$[33] \qquad h^0(\lambda) \text{ s.t. } E\left[V\left(h^0(\lambda)\right)\right] = E\left[V(\mu)\right]$$
$$(1-\lambda)V^d\left(h^0(\lambda)\right) + \lambda V^f\left(h^0(\lambda)\right) = (1-\lambda)V^d(\mu) + \lambda V^f(\mu)$$

The above equality can be rewritten as:

$$[34] \qquad \lambda\left[V^f\left(h^0(\lambda)\right) - V^f(\mu)\right] = (1\text{-}\lambda)\left[V^d(\mu) - V^d\left(h^0(\lambda)\right)\right]$$

As we have assumed that $h^{mig} < \mu$, the left hand side of [34] is positive, as V^f is increasing in its argument. This entails that $h^0(\lambda)$ has to be lower than $h^0(0)= h^0$ for every positive probability of migration. Differentiating [34] with respect to λ, we can observe that:

[19] Nothing changes if the reverse occurs.

[35] $$\frac{\partial h^0(\lambda)}{\partial \lambda} = -\frac{\left[V^f\left(h^0(\lambda)\right) - V^d\left(h^0(\lambda)\right)\right] + \left[V^d(\mu) - V^f(\mu)\right]}{\lambda\left(\frac{\partial V^f\left(h^0(\lambda)\right)}{\partial h^0(\lambda)} - \frac{\partial V^d\left(h^0(\lambda)\right)}{\partial h^0(\lambda)}\right)} < 0$$

so that the threshold value $h^0(\lambda)$ is decreasing in its argument. Thus, a positive probability λ to migrate lowers the minimal level of human capital that can be conferred to a child in his own interest. This entails that a level of parental human capital lower than h^1 suffices to finance the corresponding educational expenditure. The prospect to migrate raises the relative profitability of educational expenditure, but it does not alter the long-run dynamics of the system, as it does not suffice to move households above the critical threshold h^{in}. We now turn to the analysis of how remittances may influence the dynamic of the system.

4.2 Remittances and educational expenditure

Once an agent migrates, she sends remittances back home. A first component of remittances is represented by the eventual transfer to her parents to pay back past educational expenditure, while a second one arises from the necessity to repatriate her savings for consumption in the next period. As we have let apart the issues of non complacency with the intergenerational obligations, the migrant retains a discretional control only over the second component of the remittance transfer. The size of this discretional component depends on the saving choices of the migrant, who – according to [6] – saves a constant fraction of her disposable incomes. This savings behavior results in a tilt of consumption towards the future, to take advantage of the price differentials between the home and the foreign country, as it can be inferred from [7]; while the ratio of consumption levels in periods 2 and 3 of an agent who is domestically employed is equal to:

[36] $$\frac{c_t}{c_{t+1}} = \frac{1}{\beta(1+R)}$$

As we have ruled out domestic price inflation, the corresponding ratio for a migrant is given by:

[37] $$\frac{c_t}{c_{t+1}} = \frac{\left(1+\pi^e\right)}{\beta(1+R)} = \frac{\frac{p}{Ep^f}}{\beta(1+R)} = \frac{1}{d\beta(1+R)}$$

that is lower than the former as the real exchange rate d is greater than 1. That is, the change of residence between period 2 and 3 determines for the agent the same effects of a negative inflation rate. The discretional component of remittances are the channel through which the migrants transfer resources for their future consumption, and its size is determined by the disposable income they earn abroad. Condition [28] defined the minimal level of human capital that renders migration a profitable option, and this has an important bearing on the size of remittances. We can observe that condition [28] has the following implication for all migrants:

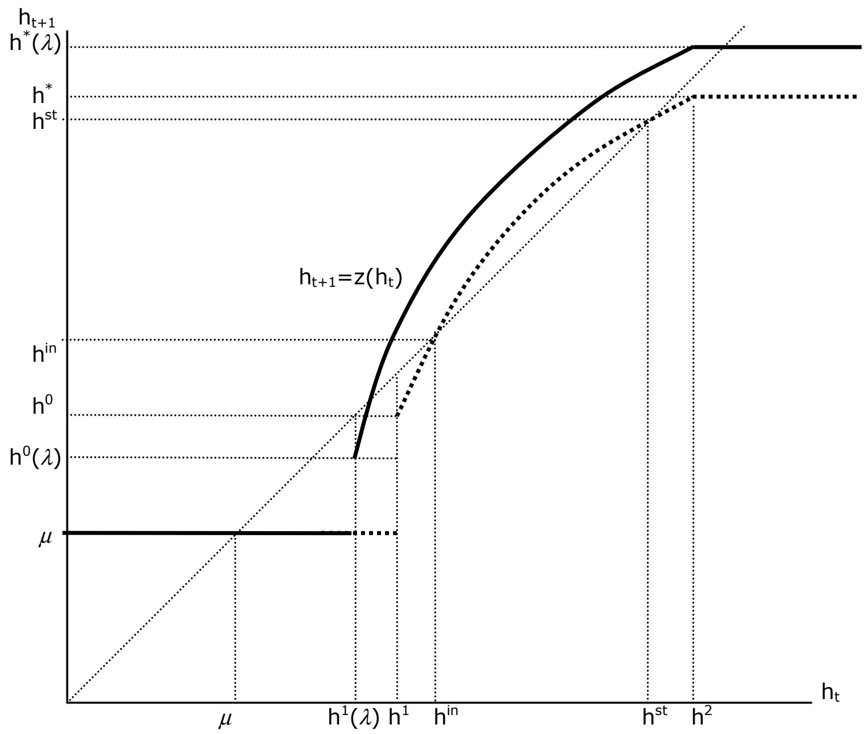
[38] $$dn^f(h_t) > d^{\frac{1}{(1+\beta)}} n(h_t) > n(h_t) \quad \text{for } h_t > h^{mig}$$

That is, the disposable income of a migrant at prices of the home country is higher than what would have been his real disposable income if she had held a domestic job. What is interesting is that also an agent who is indifferent between a domestic and a foreign employment enjoys a discrete increase in his real disposable income in domestic terms.[20] As migrants save a fixed fraction of their disposable income, condition [38] implies that they transfer – via remittances

[20] According to the optimal savings plan, as described by [6], the migrant consumes a fraction $(1+\beta)^{-1}$ of the disposable income abroad, and this is exactly the exponent of the real exchange rate d in condition [38].

– an amount of resources that is greater than their desired savings level in case of domestic employment. Thus, actual migration – for any human capital endowment of the migrant - increases the household resources that can be devoted to educational expenditure, loosening the self-financing constraint that – when binding – limits educational expenditure. While the possibility to migrate influences the relative profitability of monetary savings and educational expenditure, actual migration - via remittances – increases the resources that can be devoted to human capital formation. This implies that, for the households where the adult member holds a foreign employment, we observe an upward displacement of the curve that describes the evolution of human capital in the (h_t, h_{t+1}) space, as it is shown in Figure 2.

Figure 2. Impact of remittances on the dynamic evolution of human capital, with $\mu<h^{mig}<h^{in}$



Thus, remittances provide a boost to human capital formation in recipient households. A critical question regards the impact of remittances on the long-term dynamics of human capital formation within the household. We have assumed that migration is temporarily, so that it can produce a long-lasting impact on the household intergenerational dynamics of human capital only if it succeeds to raise the human capital of children above h^{in}, reverting the convergence towards the minimal level of human capital.

4.3 The impact of remittances on the long-term dynamics

As it has been shown in Section 3, the unstable equilibrium h^{in} plays a critical role in the long-term dynamics of human capital, as – in the absence of a possibility to migrate – a current endowment that falls short of h^{in} entails that the household will eventually decline towards μ. Migration – via remittances – provides a temporary boost to human capital investment, and it may prevent recipient households from falling into the poverty trap if it provides them sufficient resources to raise the human capital of the children above h^{in}. Such a possibility can clearly occur only if the fixed cost τ determines a threshold h^{mig} that is lower than h^{in}, as otherwise remittances can have no impact on the long-run dynamics because all recipient households would have converged to h^{st} anyway.

But, provided that $h^{mig} < h^{in}$, can recipient households escape the region of attraction of equilibrium for $h_t = \mu$? This occurs if remittances are no lower than the educational expenditure that is required to finance the accumulation of h^{in}, that is if migrants' disposable income in domestic term is no lower than the disposable income of an agent endowed with h^{in} who is domestically employed. Using [38], this condition can be formally stated as:

$$[39] \quad n\left(h_t\right) \geq d^{\frac{-1}{(1+\beta)}} n\left(h^{in}\right),\ h^{mig} < h_t < h^{in}$$

Condition [39] identifies a sufficient condition for a recipient household, i.e. $h_t > h^{mig}$, to escape the poverty trap via the receipt of remittances: its disposable income in case of domestic employment needs to be no lower than a share $d^{\frac{-1}{(1+\beta)}}$ of the disposable income in correspondence of the unstable equilibrium h^{in}. If [39] does not hold, then remittances may not suffice to generate enough resources to finance the critical investment in education. It is immediate to show that condition [39] is in general *not* satisfied for all recipients. As its left hand side is increasing in h_t^i, condition [39] holds for all recipients only if it holds as an equality for $h_t = h^{mig}$, that is if:

$$[40] \quad n\left(h^{mig}\right) = d^{\frac{-1}{(1+\beta)}} n\left(h^{in}\right)$$

We can observe that the right hand side of [40] does not depend on τ, while the left hand side does. More specifically, the left hand side is increasing in τ, as an higher cost of migration increases the minimal level of human capital that renders migration profitable. As the wage differential narrows down, that is d gets closer to 1, the right hand side of [40] approaches $n(h^{in})$. As the fixed cost τ declines to zero, h^{mig} approaches to μ, so that condition [40] clearly fails. The reverse of this argument is the following: in the absence of migration costs, any wage differential renders migration profitable for all domestic agents; but, a narrow wage differential would not produce a sufficient boost to household expenditure in education to raise the household outside the critical level of human capital.
For any wage differential, there is a non-empty left neighborhood of h^{in}, such that the recipient households whose endowments of human capital belong to that neighborhood succeed in leaving the attraction basin of μ through the increased educational expenditure that is financed via remittances. If τ is such that h^{mig} lies outside – at the left of – this neighborhood, then there will be some recipient households that will not receive remittances that allow them to increase educational expenditure sufficiently.
We can spell out a sufficient condition on the wage differential that ensures that all domestic agents, once they find it profitable to migrate, then receive sufficient remittances to move out of the attraction basin of μ:

$$[41] \quad n\left(\mu\right) = w\mu > d^{\frac{-1}{(1+\beta)}} n\left(h^{in}\right)$$

Under condition [41], also the households that are stuck at the minimal level of human capital can escape the poverty trap if they are offered the chance to migrate. Condition [41] has an intuitive interpretation, as it states that remittances represent a way out of poverty for all recipient households when the gap between the minimal endowment of human capital μ and the unstable equilibrium h^{in} is narrow enough to be overcome by the wage differential between the foreign and the home country. Thus, condition [41] depends crucially on two of the parameters of the model, that is the real exchange rate d – that depends on the real foreign w^f - and the fixed educational cost θ. Totally differentiating the right hand side of [41], we have that:

$$[42] \quad \frac{-1}{(1+\beta)} d^{\frac{-1}{(1+\beta)}-1} \partial d + d^{\frac{-1}{(1+\beta)}} \frac{\partial n\left(h^{in}\right)}{\partial \theta} \partial \theta$$

$$\frac{1}{(1+\beta)} d^{-1}\partial d = \frac{\partial n(h^{in})}{\partial\theta}\partial\theta$$

$$\frac{\partial d}{\partial\theta} = (1+\beta) d \frac{\partial n(h^{in})}{\partial\theta} > 0$$

as the disposable income $n(h^{in})$ is increasing in θ (see the appendix A4). Thus, an increase in the fixed educational cost θ widens the gap between the disposable income in correspondence of the minimal endowment of human capital and that in correspondence of h^{in}, so that condition [42] observes that this need to be offset by an increase in the wage differential reflected in d to have the condition [41] to be satisfied. A higher education cost needs to be matched by a higher income gain from migration in order to have that remittances can bring recipient households out of the poverty trap.

Once condition [41] is satisfied and the fixed cost τ entailed by migration does not exceed the threshold set by [29], then a positive probability λ to migrate can be interpreted as an equal probability of escaping from the poverty trap, as all households are willing to migrate and remittances provide enough resources to increase the household human capital endowment above h^{in}. Under [29] and [41], all households leave the basin of attraction of μ and in the long run converge towards the equilibrium h^{st}. The probability λ influences neither condition [29] nor [41], so that the effect of migration on the long-run dynamics of human capital is not dependent on the probability to migrate, as this just influences the speed of transition out of the poverty trap.[21] Under [29] and [41], the exogenous shock represented by the migration prospect to a high-wage country disrupts the poverty trap, as remittances completely alter the long-run evolution of the system.

Still, it has to be stressed that these two conditions depict a scenario that is overtly optimistic, as domestic agents are offered the opportunity to migrate at a low cost to a country where real wages are significantly higher than at home. If either of these two positive conditions fail, then the poverty trap can persist across generations notwithstanding the remittances inflow. In this case, it would be of special interest to move to the analysis of the possible indirect effects of remittances, as the persistence of a poverty trap could also be influenced by these effects.[22]

5. Limitations of the analysis and scope for future research

The specification of the model that has been analyzed so far has ruled out any influence of remittances on human capital dynamics through their general equilibrium effects, as the whole analysis has been grounded on the independence of households' dynamics. This has required to introduce hypothesis on the structure of factor markets - namely the international mobility of capital and the unrestricted domestic mobility oh human capital[23] - that render factor rewards and domestic prices constant over time. Thus, remittances could produce no effect on non recipient households, as these assumptions have excluded *a priori* any influence on the macroeconomic variables that shape the households' maximization process.[24] But this stands in sharp contrast with a strand of literature that predicts that remittances could have a significant

[21] Under [29] and [41], each period a fraction λ of the households stuck in the poverty trap moves out of it, so that the higher the probability to migrate, the more rapid will be the increase of the number of households that converge towards the efficient equilibrium.

[22] However – as it is described in section 5 - such a move is not just called for by the observation that direct effects alone may not suffice to alter the inter-generational dynamics of human capital, as the economic literature on migration and remittances does not allow to assume *a priori* these effects to be positive.

[23] Since workers are assumed to work for just one period in their lives, this hypothesis is rather natural, as strictly speaking workers never move across sectors, as they choose just once their sector of employment. A departure from this hypothesis requires to assume that workers have an incentive to work in the sector where their parents have been employed, because, say, a part of the human capital they inherit is sector-specific, so that it would be lost in case of employment in the other sector.

[24] Migration reduces the aggregate endowment of human capital, but an accommodating outflow of capital maintains the ratio between the two factors unchanged, so that the wage and the profit rate do not vary; remittances increase the demand for both the traded and the non traded good, but the ensuing upward pressure on the price of the latter is fully offset by the intersectoral movement of the labor force towards the non traded sector and out of the traded sector, so that domestic prices do not increase with remittances. Under these assumptions, remittances just increase the size of the domestic non traded sector, but this produces no additional effect.

impact precisely on these variables, and an impact that needs not to favour human capital formation in non recipient households. This suggests the opportunity to relax the assumptions on factor markets, as these may be clog some relevant channels through which migration and remittances could alter the aggregate dynamics of human capital. Thus, this section describes the theoretical and the empirical arguments that draw the attention on the limitations of our model and that call for its extension, although this remains beyond the scope of this paper, that nevertheless provides an analytical structure that lays the ground for introducing general equilibrium effects of remittances.[25]

The influential paper by Corden and Neary (1982) on the so called *Dutch Disease* has inspired a strand of literature on remittances, that predicts that a consistent flow of workers' remittances could produce effects that are similar to those arising from a natural resource boom. Remittances can have an uneven sectoral impact, increasing the demand for non traded goods, moving resources towards the non traded sector and igniting inflationary pressures that lead to an appreciation of the real exchange rate.[26] From a theoretical standpoint, these models assume labor to be homogeneous and migration and remittances decisions to be exogenous – with McCormick and Wahba (2000) representing an exception, so that they do not consider eventual micro implications of the macro effects of remittances. Rivera-Batiz (1986) and Quibria (1997) suggest that remittances reinforce the inflationary pressures determined by labor migration, under the assumptions that the non tradable good sector is labor intensive and that the stock of capital is given. The shift in the relative sectoral price has a bearing on factor rewards, that could lead non recipient households to benefit from remittances (Quibria 1997, Djajić 1998). In the words of Rivera-Batiz (1986), "non-migrant welfare can be affected by emigration only in so far as it changes non-traded goods prices".

McCormick and Wahba (2000) maintain the assumption of the international immobility of capital,[27] while they differ from previous model as they introduce heterogeneity in the domestic workforce and endogenize the decision to migrate and remit; workers can be either educated or uneducated – with an exogenously given proportion between the two groups - and this heterogeneity is matched by a segmentation of the labor market, as only educated workers can access the high-wage urban traded sector. Uneducated workers can be employed either in the rural traded sector, where they earn the average product per worker, or in the urban non traded sector. Educated workers that do not find employment in the urban traded sector can freely decide to migrate, while just a fixed quota of uneducated workers is admitted in the destination country. Migration - and remittances - raise the domestic price of non traded goods, but this shift in relative price has no bearing on wages as uneducated workers are assumed to move freely between rural and urban areas, so that the wage in the non traded sector is determined by the average productivity in the agricultural sector. Thus, remittances depress the domestic wages, and non recipient households that rely just on labor income are adversely affected. While Quibria (1997), Rivera-Batiz (1986) and Djajić (1998) predict that remittances increase the real incomes of non recipient households if capital does not promptly adjust to the migration of labor, McCormick and Wahba (2000) suggests that this does not happen if nominal wages are anchored to the average productivity in the agricultural traded sector and so do not keep up with the increase of domestic prices consequent on remittances.

These models suggest that alternative hypothesis on the structure of factor markets could determine a rich array of indirect impact of remittances on non recipient households, so that the tentative implications that have been derived in this model represent a first step that needs to be confronted with further research. This research will have to deal with the analytical problems posed by the removal of the strict assumption of the independence of households' dynamics.

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[25] Note that one could also introduce an intra-generational externality from educational investment, as in Rapoport and Docquier (2005); the analytical challenges arise from the interdependence of households' dynamics rather then from the underlying reasons of such an interdependence.

[26] The empirical evidence in this respect is limited to a handful of studies, but, although far from being conclusive, it is nevertheless supportive of the theoretical argument that remittances induce a real appreciation; Amuedo-Dorantes and Pozo (2004) find a significant and negative elasticity of the real exchange rate with respect to remittances for Latin American countries, a conclusion that has been recently supported by Acosta et al. (2006) and World Bank (2006) for the same region; Bourdet and Falck (2006) reach a similar conclusion for Cape Verde, while Rajan and Subramanian (2005) do not find support for the idea that remittances could give rise to a *Dutch Disease*.

[27] Djajić (1998) assumes foreign capital to be domestically employed, but in a given quantity that is not sensitive to either migration and remittances.

6. Conclusions

This paper develops an overlapping generation model that builds on the analysis by Rapoport and Docquier (2005) around the impact of remittances on human capital formation. The model, where parents invest in the education of their children for weakly altruistic motivations (Brown and Poirine 2005), is characterized by the existence of multiple equilibria in the inter-generational dynamics of human capital; these arise because of local non-convexities in educational investments and because of a credit market failure that obliges households to finance education costs just out of their current income. We then introduce an exogenous probability to migrate – at a given fixed cost τ - to a high-wage country, to assess how the human capital dynamics is influenced by the remittances sent by the migrants. The existence of a fixed migration cost entails that – for any wage differential - it exists a critical threshold of human capital such that a domestic agent is willing to accept a foreign wage only if he is endowed with a human capital in excess of this threshold. Remittances provide a boost to educational expenditures in recipient households, as even the agent who is marginally indifferent between the domestic and the foreign job raises his investment in education by a discrete, positive, amount. This temporary increase in educational expenditure can have long-lasting consequences on inter-generational dynamics if it suffices to drive the household endowment of human capital out of the attraction basin of the poverty trap. Under favourable hypothesis on the size of the wage differential and on the migration costs, the exogenous probability to migrate acts just like an equal probability to move out of the poverty trap. In the long run, this fades away, as all households converge to the unique stable equilibrium at a low level of human capital. Still, this conclusion rests on overtly optimistic assumption of a wide wage differential and of a low migration cost, and it disregards the general equilibrium effects of remittances, as the entire analysis is built on the hypothesis of the independence of households' decision problems. This latter hypothesis appears to be extremely restrictive, and it needs to be removed in a future analysis that will be based on the grounds laid in this paper.

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Appendix

A1. Equilibrium on the goods market

Both goods are assumed to be non storable, so that by the definition of a non tradable good its production and consumption needs to match in every period:

[a1] $\widehat{c_t^N} = g_t^N$, where $\widehat{c_t^N} = \sum c_t^N$

On the tradable goods market, instead, domestic production and consumption do not to need to meet. First, capital is assumed to be foreign owned, so that a share α of the domestic production is transferred abroad as a factor reward. Second, agents can save in the second period of their lives and dissave in the third, and this can drive a further wedge between current production and consumption. Call M_t the total amount of interest-bearing assets accumulated by the economic agents - as agents cannot borrow, this is certainly non negative.[28]

[a2] $(1-\alpha)g_t^T = \widehat{c_t^T} + \frac{M_t}{p_t^T}$, where $\widehat{c_t^T} = \sum c_t^T$

Nominal national income at time t is equal to the product between the nominal wage W_t and the economy-wide endowment of human capital $\widehat{h_t}$ that is domestically employed, plus the current value of time t-1 monetary savings M_{t-1} and the domestic currency value of remittances received from abroad, D_t. Note that national income is insensitive to changes in the intersectoral allocation of human capital, as the wage rate is constant across sectors. Given the homotheticity of agents' preferences in consumption, the demand levels of the two goods are equal to:

[a3] $\widehat{c_t^N} == (1-q_T)\frac{\left[W_t\widehat{h_t} + (1+R)M_{t-1} + D_t\right]}{p_t^N}$, $\widehat{c_t^T} == q_T\frac{\left[W_t\widehat{h_t} + (1+R)M_{t-1} + D_t\right]}{p_t^T}$

From [a3], we can derive the relationship between the demand of the two goods:

[a4] $\widehat{c_t^T} = \frac{q_T p_t^N}{(1-q_T)p_t^T}\widehat{c_t^N} = \frac{q_T}{(1-q_T)}(a_t)^{-1}\left(\frac{\partial g(h_t^T, K_t)}{\partial h_t^T}\right)\widehat{c_t^N}$

As the relative price a_t is fully determined by the equilibrium on the labor market, we have that the demand levels of the two goods are fixed, and the equilibrium has to be attained through variations on the production side, that is to say with changes in the sectoral allocation of human capital. Given the equilibrium condition on the non tradable good market, the tradable good market is in equilibrium if and only if:

[a5] $g_t^T = \frac{q_T}{a_t(1-q_T)(1-\alpha)}g_t^N + \frac{M_t}{(1-\alpha)p_t^T}$

This requires h_t^T, the total human capital employed in the tradable sector, to satisfy:

[a6] $g(h_t^T, K_t) = \frac{q_T}{a_t(1-q_T)(1-\alpha)}\left(\widehat{h_t} - h_t^T\right) + \frac{M_t}{(1-\alpha)p_t^T}$

[28] As we have assumed that both goods are non storable, this entails that domestic agents are lending abroad.

The endowment of physical capital adjusts to sectoral shifts of human capital to ensure that the profit rate stays put at R and the wage rate per unit of human capital remains constant at W_t. Thus, labor income is independent of the sectoral allocation of labor, and thus it is the level of monetary savings. A value of h_t^T that satisfies [a6] always exist; note first that the left hand side of [a6] is increasing in h_t^T, while the right hand side is decreasing. To demonstrate that an equilibrium value of h_t^T exists we just need that to observe that both sides of [a6] are continuous in h_t^T and that:

[a7] $$\lim_{h_t^T \to 0} g\left(h_t^T, K_t\right) = 0 < \lim_{h_t^T \to 0} \frac{q_T}{a_t\left(1-q_T\right)\left(1-\alpha\right)}\left(\widehat{h_t} - h_t^T\right) + \frac{M_t}{\left(1-\alpha\right)p_t^T} = \frac{q_T}{a_t\left(1-q_T\right)\left(1-\alpha\right)}\widehat{h_t} + \frac{M_t}{\left(1-\alpha\right)p_t^T}$$

[a8] $$\lim_{h_t^T \to \widehat{h_t}} g\left(h_t^T, K_t\right) = \frac{\widehat{h_t}}{a_t} > \lim_{h_t^T \to \widehat{h_t}} \frac{q_T}{a_t\left(1-q_T\right)\left(1-\alpha\right)}\left(\widehat{h_t} - h_t^T\right) + \frac{M_t}{\left(1-\alpha\right)p_t^T} = \frac{M_t}{\left(1-\alpha\right)p_t^T}$$

A2. Non profitability of educational expenditure at $h_t = \mu$.

A poverty trap emerges in the intergenerational dynamics of human capital if educational expenditure is not profitable at $h_t = \mu$. Intuitively, this depends on the height of the fixed cost entailed by educational expenditure. We need to derive the restrictions on the parameters that ensure that.

[a9] $$w\left[\ln\left(\frac{\beta}{1+\beta}w\mu - \theta + \nu\right) - \mu\right] < \left(1+R\right)\left(\frac{\beta}{1+\beta}w\mu\right)$$

With some simple algebra, condition [a9] can be rearranged as follows:

[a10] $$\ln\left(\frac{\beta}{1+\beta}w\ln\left(\nu\right) - \theta + \nu\right) - \ln\left(\nu\right) < \left(1+R\right)\left(\frac{\beta}{1+\beta}\ln\left(\nu\right)\right)$$

$$\ln\left(\frac{\beta}{1+\beta}w\ln\left(\nu\right) - \theta + \nu\right) < \left(\frac{\beta\left(1+R\right)+\left(1+\beta\right)}{1+\beta}\ln\left(\nu\right)\right)$$

$$\frac{\beta}{1+\beta}w\ln\left(\nu\right) - \theta + \nu < \nu^{\frac{\beta(1+R)+(1+\beta)}{1+\beta}}$$

$$\theta > \theta^1 = \nu + \frac{\beta}{1+\beta}w\ln\left(\nu\right) - \nu^{\frac{\beta(1+R)+(1+\beta)}{1+\beta}}$$

As long as $\theta > \theta^1$, educational expenditure is not profitable at $h_t = \mu$, as its average return falls short of its average – and marginal – cost. By definition of h^0 and h^1, this entails that educational investment is also not profitable is the parental endowment of human capital is lower than h^1. For what follows, it is interesting to observe how the restriction θ^1 on the fixed cost changes with $\nu = e^{\mu}$. Let us define:

[a11] $$j(\nu) = \theta^1 - \nu = \frac{\beta}{1+\beta}w\ln\left(\nu\right) - \nu^{\left[\frac{\beta(1+R)+(1+\beta)}{(1+\beta)}\right]}$$

The derivative of $j(\nu)$ with respect to its argument is equal to:

[a12] $$\frac{\partial j\left(\nu\right)}{\partial \nu} = \nu^{-1}\left(\frac{\beta w}{1+\beta} - \frac{\beta\left(1+R\right)+\left(1+\beta\right)}{\left(1+\beta\right)}\nu^{\left[\frac{\beta(1+R)+(1+\beta)}{(1+\beta)}\right]}\right)$$

and it can be verified by visual inspection of [a12] that the second order derivative of j(ν) is negative. Thus, the function j(ν) attains its maximum for $\nu = \nu^M$, where ν^M is the value that renders its derivative equal to 0. Moving around the terms between parenthesis in [a12], it is immediate to show that ν^M is equal to:

$$\text{[a13]} \quad \nu^M = \left(\frac{\beta w}{\beta(1+R)+(1+\beta)}\right)^{\frac{(1+\beta)}{\beta(1+R)+(1+\beta)}}$$

If we compute the value of j(ν) in correspondence of $\nu = \nu^M$ we obtain:

$$\text{[a14]} \quad j(\nu^M) = \frac{w}{\gamma}\left[\ln\left(\frac{w}{\gamma}\right) - 1\right], \text{ where } \gamma = \frac{\beta(1+R)+(1+\beta)}{\beta}.$$

This entails that θ^1 is not higher than $j(\nu^M)+\nu$ for any value of ν, that is:

$$\text{[a15]} \quad \theta^1 \le \nu + j(\nu^M) = \nu + \frac{w}{\gamma}\left[\ln\left(\frac{w}{\gamma}\right) - 1\right]$$

A3. Additional equilibria in the dynamic system

Under [a15], a poverty trap emerges; the model has an analytical interest if the dynamic system $h_{t+1} = z(h_t)$ presents additional equilibria towards which household can converge, that is there are alternatives to the poverty trap. Additional equilibria happen to exist under the restrictions that ensure that in correspondence of the point $h^§$ where the function $z(h_t)$ has a unitary derivative, we have that $z(h^§) > h^§$, that is:

$$\text{[a16]} \quad z(h^§) > h^§ \text{ for } h^§ \text{ s.t. } \left.\frac{\partial z(h_t)}{\partial h_t}\right|_{h_t = h^§} = 1$$

The function $z(h_t)$ is continuous for $h_t > h^1$, and thus [a10] and [a16] together imply that there must be a point h^{in} comprised between h^1 and $h^§$ such that $z(h^{in}) = h^{in}$. Moreover, as the function $z(h^t)$ is concave for $h^1 < h_t < h^2$, we have that this equilibrium is unstable, as it lies at the left of the point $h^§$ that is by definition characterized by a unitary derivative. Under [a16], it also follows that the system presents a third, locally stable, equilibrium. As said, the function $z(h_t)$ is continuous for $h_t > h^1$ and it is upper bounded, so that under [a16] there is necessarily a third equilibrium point at the right of $h^§$, that can be shown to be lower than h^*, the optimal level of human capital that is not sustainable across generations. We thus need to derive the parametric restrictions that ensure that [a16] holds; the derivative of $z(h^t)$ for $h^1 < h_t < h^2$ is given by:

$$\text{[a17]} \quad \frac{\partial z(h_t^i)}{\partial h_t^i} = \frac{1}{e^{z(h_t^i)}}\left(\frac{\beta}{1+\beta}\left[w - (1+R)e^{h_t^i}\right]\right)$$

With some straightforward algebra but tedious algebra, we can derive the value of $h^§$:

$$\text{[a18]} \quad \frac{1}{e^{z(h^§)}}\left(\frac{\beta}{1+\beta}\left[w - (1+R)e^{h^§}\right]\right) = 1$$

$$e^{z(h^§)} = \left(\frac{\beta}{1+\beta}\left[w - (1+R)e^{h^§}\right]\right)$$

$$\frac{\beta}{1+\beta}\left[wh^{\S}-(1+R)\left(e^{h^{\S}}+\theta-\nu\right)\right]-\theta+\nu=\frac{\beta}{1+\beta}\left[w-(1+R)e^{h^{\S}}\right]$$
$$-\frac{\beta(1+R)+(1+\beta)}{1+\beta}(\theta-\nu)+\frac{\beta}{1+\beta}wh^{\S}=\frac{\beta}{1+\beta}w$$
$$wh^{\S}=w+\frac{\beta(1+R)+(1+\beta)}{\beta}(\theta-\nu)$$
$$h^{\S}=1+\frac{\gamma(\theta-\nu)}{w}$$

To meet condition [a16], we then need to have that:

$$\text{[a19]}\quad e^{z\left(h^{\S}\right)}=\frac{\beta}{1+\beta}\left[wh^{\S}-(1+R)\left(e^{h^{\S}}+\theta-\nu\right)\right]-\theta+\nu>e^{h^{\S}}$$

According to [a18], the above inequality can be rewritten as:

$$\text{[a20]}\quad \frac{\beta}{1+\beta}\left[w-(1+R)e^{h^{\S}}\right]>e^{h^{\S}}$$
$$\frac{\beta}{1+\beta}w>\frac{\beta(1+R)+(1+\beta)}{1+\beta}e^{h^{\S}}$$
$$e^{h^{\S}}<\frac{w}{\gamma}$$
$$h^{\S}<\ln\left(\frac{w}{\gamma}\right)$$

Combining condition [a18] with [a20], we can introduce the further restriction on θ we are looking for:

$$\text{[a21]}\quad h^{\S}=1+\frac{\gamma(\theta-\nu)}{w}<\ln\left(\frac{w}{\gamma}\right)$$
$$\theta<\theta^{2}=\nu+\frac{w}{\gamma}\left[\ln\left(\frac{w}{\gamma}\right)-1\right].$$

As long as $\theta < \theta^2$, condition [a16] is always satisfied, and the dynamic system presents two additional equilibria. It is straightforward to observe that $\theta^1 < \theta^2$, i.e. the two restrictions are compatible, as long as $\nu \neq \nu^M$, as this entails that:

$$\text{[a22]}\quad \theta^{1}<\nu+j\left(\nu^{M}\right)=\nu+\frac{w}{\gamma}\left[\ln\left(\frac{w}{\gamma}\right)-1\right]=\theta^{2}$$

The restrictions that ensure that the dynamic system displays three distinct equilibria are:

$$\text{[a23]}\quad \theta^{1}=\nu+\frac{\beta}{1+\beta}w\ln(\nu)-\nu\frac{\beta(1+R)+(1+\beta)}{1+\beta}<\theta<\nu+\frac{w}{\gamma}\left[\ln\left(\frac{w}{\gamma}\right)-1\right]=\theta^{2},\ \ \nu\neq\nu^{M}.$$

It is straightforward to show that under [a23] $h^* < h^2$, that is the optimal level of human capital is not sustainable across generations, and the third equilibrium of the system lies at the left of h^*. To prove this, it suffices that under [a23] the savings of an agent who is endowed with h^* fall short the optimal educational expenditure, that is:

$$\text{[a24]}\quad \frac{\beta}{1+\beta}\left[w\ln\left(\frac{w}{1+R}\right)-(1+R)\left(\frac{w}{1+R}+\theta-\nu\right)\right]<\left(\frac{w}{1+R}+\theta-\nu\right)$$

With some simple algebra, the above inequality can be rewritten as follows:

[a25] $$\frac{\beta}{1+\beta}\left[w\ln\left(\frac{w}{1+R}\right)-(1+R)\left(\frac{w}{1+R}+\theta-\nu\right)\right]<\left(\frac{w}{1+R}+\theta-\nu\right)$$
$$w\ln\left(\frac{w}{1+R}\right)<\frac{\beta(1+R)+(1+\beta)}{\beta}\left(\frac{w}{1+R}+\theta-\nu\right)$$
$$\theta<\frac{w}{\gamma}\ln\left(\frac{w}{1+R}\right)-\frac{w}{1+R}+\nu$$

We can now demonstrate that $\theta < \theta^2$ implies that [a25] is met, and thus the optimal level of human capital is not sustainable:

[a26] $$\frac{w}{\gamma}\ln\left(\frac{w}{1+R}\right)-\frac{w}{1+R}+\nu<\nu+\frac{w}{\gamma}\left[\ln\left(\frac{w}{\gamma}\right)-1\right]=\theta^2$$
$$\ln\left(\frac{w}{1+R}\right)-\frac{\gamma}{1+R}<\ln\left(\frac{w}{\gamma}\right)-1$$
$$\ln\left(\frac{\gamma}{1+R}\right)<\frac{\gamma}{1+R}-1$$

that is certainly satisfied as $\gamma > (1+R)$.

A4. Shifts of the equilibria in response to a variation in the fixed educational cost θ.

Under [a23], there are two equilibria h^{in} and h^{st} for $h^1 < h_t < h^2$. We can observe how these equilibria shift in response to a variation in the parameter θ that identifies the fixed cost in education. First, we can observe that:

[a27] $$h_{t+1}=\frac{\partial z(h_t)}{\partial h_t}=\frac{w-(1+R)e^{h_t}}{\left[wh_t-(1+R)e^{h_t}-\gamma(\theta-\nu)\right]} \quad \text{for } h^1<h_t<h^2.$$

We have that this derivative is greater than 1 in correspondence of h^{in} and lower than 1 in correspondence of h^{st}, so that the two equilibrium are such that:

[a28] $$w\left(h^{in}-1\right)-\gamma(\theta-\nu)>0 \quad \text{and} \quad w\left(h^{st}-1\right)-\gamma(\theta-\nu)<0$$

In correspondence of both equilibria, by definition, we have that $h_t = z(h_t)$, that is:

[a29] $$h_t=\ln\left(\frac{\beta}{1+\beta}\left[wh_t-(1+R)\left(e^{h_t}+\theta-\nu\right)\right]-\theta+\nu\right)=\ln\left(\frac{\beta}{1+\beta}\right)+\ln\left(wh_t-(1+R)e^{h_t}-\gamma(\theta-\nu)\right)$$

Totally differentiating the above equality with respect to h_t and θ, we can derive the relationship between the fixed education cost and the position of the two equilibria, as we must have that:

[a30] $$\partial h_t-\frac{w-(1+R)e^{h_t}}{\left(wh_t-(1+R)e^{h_t}-\gamma(\theta-\nu)\right)}\partial h_t+\frac{\gamma}{\left(wh_t-(1+R)e^{h_t}-\gamma(\theta-\nu)\right)}\partial\theta=0$$
$$\frac{\partial h_t}{\partial\theta}=\frac{\gamma}{w(h_t-1)-\gamma(\theta-\nu)}$$

According to [a28], condition [a30] entails that:

$$[a31]\quad \left.\frac{\partial h_t}{\partial\theta}\right|_{h_t=h^{in}} = \frac{\gamma}{w\left(h^{in}-1\right)-\gamma(\theta-\nu)} > 0, \quad \left.\frac{\partial h_t}{\partial\theta}\right|_{h_t=h^{st}} = \frac{\gamma}{w\left(h^{st}-1\right)-\gamma(\theta-\nu)} < 0.$$

Thus, an increase in the fixed educational cost entails an increase in the unstable equilibrium, and a decrease in the stable one. Once θ approaches its uppers bound θ^2, the two equilibria converge. Conversely, the closer is θ to $\theta^{\,1}$, the closer the unstable equilibrium gets to the minimal endowment of human capital μ. Consider now how the agent's disposable equilibrium in correspondence of h^{in}, $n(h^{in})$, varies in response to a shift in θ. A first direct effect is due to the change in the fixed educational cost, and a second one operates via the value of h^{in}, and the two have contrasting effects, as the disposable income in increasing in its argument for $\mu < h^t < h^*$, while it is clearly decreasing in the fixed educational cost. We can show that the former effect is prevailing, so that the $n(h^{in})$ is increasing in θ. We have that:

$$[a32]\quad \left.\frac{\partial n(h_t)}{\partial\theta}\right|_{h_t=h^{in}} = w\frac{\partial h^{in}}{\partial\theta} - (1+R)e^{h^{in}}\frac{\partial h^{in}}{\partial\theta} - (1+R)$$

If we insert [a31] in [a32], we obtain:

$$[a33]\quad \left.\frac{\partial n(h_t)}{\partial\theta}\right|_{h_t=h^{in}} = \frac{\gamma\left[w-(1+R)e^{h^{in}}\right]}{w\left(h^{in}-1\right)-\gamma(\theta-\nu)} - (1+R) = \frac{\gamma\left[w-(1+R)e^{h^{in}}\right]-(1+R)\left[w\left(h^{in}-1\right)-\gamma(\theta-\nu)\right]}{w\left(h^{in}-1\right)-\gamma(\theta-\nu)}$$

As the denominator in [a33] is negative, we can focus just on its numerator to demonstrate that [a32] is indeed positive. With some simple algebra, we can observe that the numerator is positive if:

$$[a34]\quad \gamma\left[w-(1+R)e^{h^{in}}\right]-(1+R)\left[w\left(h^{in}-1\right)-\gamma(\theta-\nu)\right] > 0$$

$$\frac{w}{(1+R)} > e^{h^{in}} + \frac{w}{\gamma}\left(h^{in}-1\right)-(\theta-\nu)$$

$$e^{h^*} > e^{h^{in}} + \frac{w}{\gamma}\left(h^{in}-1\right)-(\theta-\nu)$$

As we know that h^{in} is lower then $h^{\S}$, we can substitute the value of the latter from [a18] for h^{in}:

$$[a35]\quad e^{h^*} > e^{h^{in}} + \frac{w}{\gamma}\left(1+\frac{\gamma(\theta-\nu)}{w}-1\right)-(\theta-\nu) = e^{h^{in}}$$

that is certainly satisfied, so that we can conclude that $\frac{\partial n\left(h^{in}\right)}{\partial\theta} > 0$.

A5. The migration choice

We define as $V^d(h_t)$ the life-time utility of an agent who is endowed with h_t units of human capital, holds a domestic job and chooses the savings plan that maximizes [5]. Similarly, we define as $V^f(h_t)$ the life-time utility of an agent who holds a foreign job in the second period of her life, returns home once she retires and chooses the savings plan that maximizes [5]. Her real disposable income is labelled as $n^f(h_t)$, and it is defined extending the definition of $n(h_t)$ provided in [21], that is:

$$[a36]\quad n^f(h_t) = \begin{cases}\left(w^f h_t - \tau d^{-1} - (1+R)\,d^{-1}\left(e^{h_t}+\theta-\nu\right)\right) & \text{if } h_t > h^0 \\ w^f\mu - \tau d^{-1} & \text{otherwise}\end{cases}$$

As the optimal savings plan – as described by [6] – implies that each agent saves a constant fraction of the disposable income in period 2, $V^d(h_t)$ and $V^f(h_t)$ can be expressed as follows:

[a37] $V^d(h_t) = \ln\left[pn(h_t)\right]^{(1+\beta)} - \ln(p)^{(1+\beta)} + B$, where $B = \ln\left(\frac{q}{1+\beta}\right) + \beta\ln\left(\frac{q\beta}{1+\beta}\right)$

$$V^f(h_t) = \ln\left[p^f n^f(h_t)\right]^{(1+\beta)} - \ln\left(p^f\right) - \beta\ln\left(\frac{E}{p}\right) + B.$$

It is straightforward to observe that:

[a38] $$\forall h_t : \frac{\partial V^d(h_t)}{\partial h_t} < \frac{\partial V^f(h_t)}{\partial h_t}$$

as the life-time utility is monotonically increasing in the second period disposable income, and $w^f > w$ and $d > 1$. When $h_t > \mu$, $V^d(h_t)$ and $V^f(h_t)$ are equal to:

[a39] $$V^d(h_t) = \ln\left[Wh_t - (1+R)p\left(e^{h_t} + \theta - \nu\right)\right]^{(1+\beta)} - \ln(p)^{(1+\beta)} + B,$$

$$V^f(h_t) = \ln\left[W^f h_t - \frac{p}{E}\tau - (1+R)\frac{p}{E}\left(e^{h_t} + \theta - \nu\right)\right]^{(1+\beta)} - \ln\left(p^f\right) - \beta\ln\left(\frac{E}{p}\right) + B.$$

Migration represents a profitable choice as long as:

[a40] $$V^f(h_t) > V^d(h_t)$$

Substituting [a39] in the above inequality, this can be shown to be equivalent to:

[a41] $$\ln\left[\frac{W^f h_t - \frac{p}{E}\tau - (1+R)\frac{p}{E}\left(e^{h_t} + \theta - \nu\right)}{Wh_t - (1+R)p\left(e^{h_t} + \theta - \nu\right)}\right]^{(1+\beta)} > \ln\left(p^f\right) - \beta\ln(E) - \ln(p)$$

If we add $(1+\beta)\left[\ln(p) - \ln\left(p^f\right)\right]$ to both sides of [a41], using the properties of logarithms we can rewrite the above condition as follows:

[a42] $$\ln\left[\frac{w^f h_t - \frac{p}{Ep^f}\tau - (1+R)\frac{p}{Ep^f}\left(e^{h_t} + \theta - \nu\right)}{wh_t - (1+R)\left(e^{h_t} + \theta - \nu\right)}\right]^{(1+\beta)} > \ln\left(\frac{p}{Ep^f}\right)^{\beta}$$

$$\frac{w^f h_t - \frac{p}{Ep^f}\tau - (1+R)\frac{p}{Ep^f}\left(e^{h_t} + \theta - \nu\right)}{wh_t - (1+R)\left(e^{h_t} + \theta - \nu\right)} > \left(\frac{p}{Ep^f}\right)^{\frac{\beta}{(1+\beta)}}$$

Condition [a42] implicitly defines a threshold level of human capital, that we label as h^{mig}, such that an agent is willing to migrate only if he his endowed with a human capital in excess of h^{mig}. Such a threshold is defined as:

[a43] h^{mig} s.t. $$\left.\frac{\left(w^f h_t - \frac{p}{Ep^f}\tau - \frac{p}{Ep^f}(1+R)\left(e^{h_t}+\theta-\nu\right)\right)}{\left(wh_t^i - (1+R)\left(e^{h_t}+\theta-\nu\right)\right)}\right|_{h_t=h^{mig}} = \left(\frac{p}{Ep^f}\right)^{\frac{\beta}{(1+\beta)}}$$

Totally differentiating [a43] it can be shown that h^{mig} is declining in the wage differential and increasing in the cost τ entailed by migration. If h^{mig} is lower than h^0, then all agents whose parents have invested in education are willing to migrate. Still, this does not suffice to ensure that also the agents that have the minimal endowment of human capital share the same willingness, as this requires that:

[a44] $$\frac{w^f\mu - \frac{p}{Ep^f}\tau}{w\mu} > \left(\frac{p}{Ep^f}\right)^{\frac{\beta}{(1+\beta)}}$$

that may fail even though $h^{mig}<h^0$, as the foregone domestic disposable income is by definition the same for the agents endowed with either h^0 and μ, but the income gains from migration are increasing with h_t, so that migration can be profitable at h^0 but not at μ. Moving around the terms of [a44], we can show that all domestic agents are willing to migrate only if w^f and τ satisfy the following inequality:

[a45] $$\tau < \left[\left(\frac{w^f}{w}\right)^{\frac{1}{q_T}} - \left(\frac{w^f}{w}\right)^{\frac{(1-q_T)}{(1+\beta)q_T}}\right] w\mu$$

For a given wage differential between the two countries, condition [a45] imposes an upper threshold on the migration costs; if [a45] fails, then the agents that are endowed with μ do not find profitable to migrate, as the fixed cost τ is too high and it offsets the prevailing wage differential.